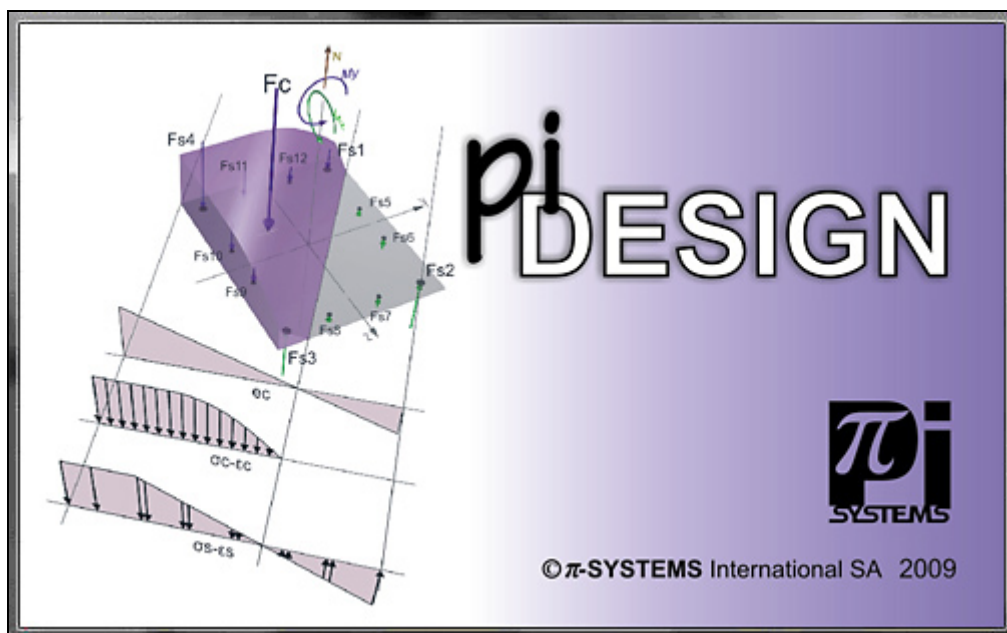


pi DESIGN

Scientific Documentation of software for
flexural design according to EC2



Copyright: Apostolos Konstantinidis & pi-SYSTEMS INTERNATIONAL SA
 4 Fokianou Str., GR-11635, ATHENS
 Tel. +30 210 7569600 (10 lines), FAX +30 210 7569609
 e-mail: info@pi.gr

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Scientific Documentation of software for flexural design according to EC2



ATHENS
JULY 2010

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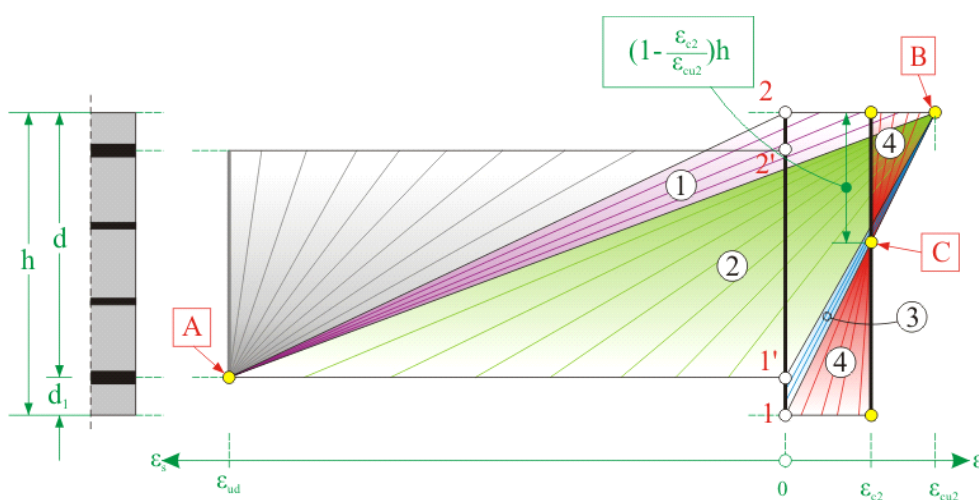
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ULTIMATE LIMIT STATE OF FAILURE DUE TO FLEXURE WITH OR WITHOUT AXIAL FORCE

1. General

For the determination of flexural capacity of reinforced concrete cross-sections the following assumptions are made:

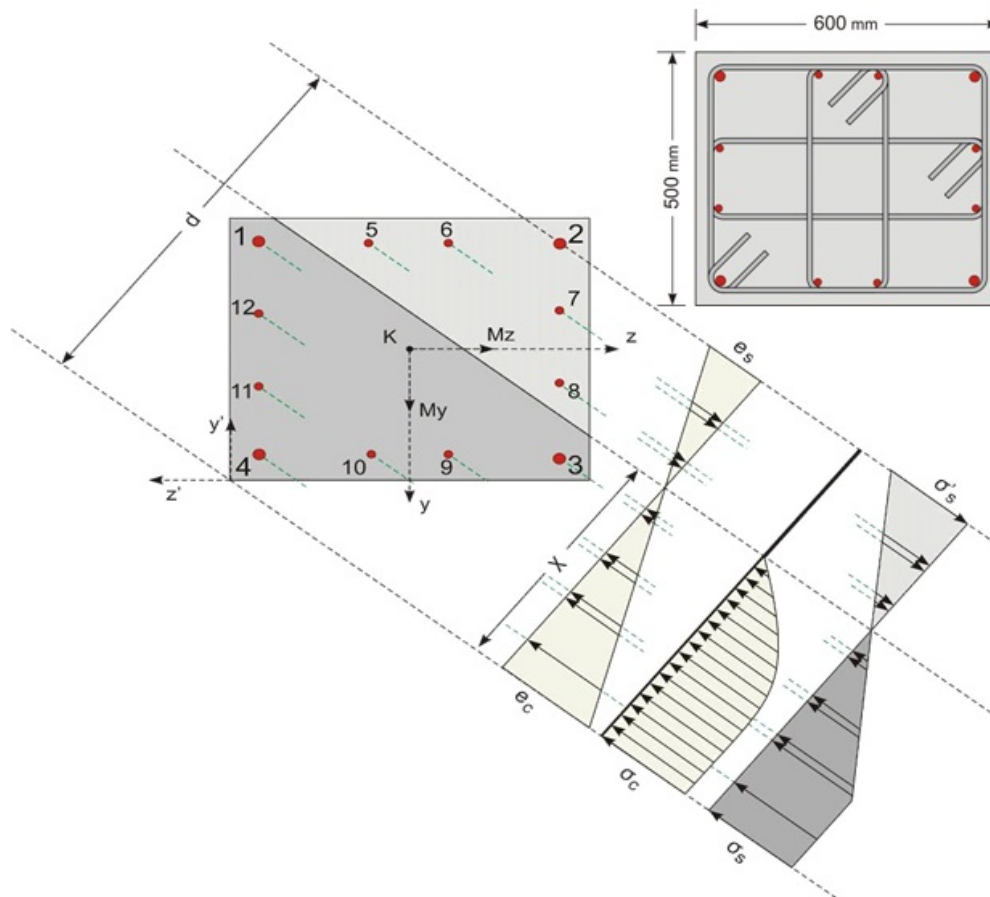
- plane sections remain plane after loading,
- the strain in reinforcement under tension or compression is the same as that in the surrounding concrete,
- the tensile strength of concrete is omitted,
- the compressive strain of concrete should not exceed ϵ_{cu2} ($\epsilon_{cu2} = -3.5\text{‰}$ for concrete strength classes $\leq C50$) and the reinforcement strain should not exceed ϵ_{ud} ($\epsilon_{ud} = 67.5\text{‰}$ for steel ductility class C),
- in case the cross-section is fully under compression, average compressive strain in concrete is not allowed to exceed ϵ_{c2} ($\epsilon_{c2} = -2.0\text{‰}$ for concrete strength classes $\leq C50$)
- The domain of allowable distribution of strains is shown on the following figure.
- In each stress condition there are two possibilities: either one part of the cross-section is under compression or the cross-section is fully under compression.



Allowable strain distributions under limit state of failure

- When only a portion of the cross-sectional area is under compression, it is assumed either that the ultimate strain of reinforcement steel ϵ_{ud} (point A) or the ultimate strain in concrete ϵ_{cu2} (point B) is reached.
 - When the ultimate strain of reinforcing steel ϵ_{ud} is reached, strain of concrete varies between 0 and ϵ_{cu2} [area 1].
 - When the ultimate strain of concrete ϵ_{cu2} is reached, strain of steel varies between 0 and ϵ_{ud} [area 2].
 - When a part of the cross-sectional area between the bottom of the section and the lowest reinforcement layer is under tension, it is assumed that the ultimate strain of concrete ϵ_{cu2} is reached [area 3].
- When all cross-sectional area is under compression, it is assumed that the distribution of strains lies on the line that passes from point C [area 4].

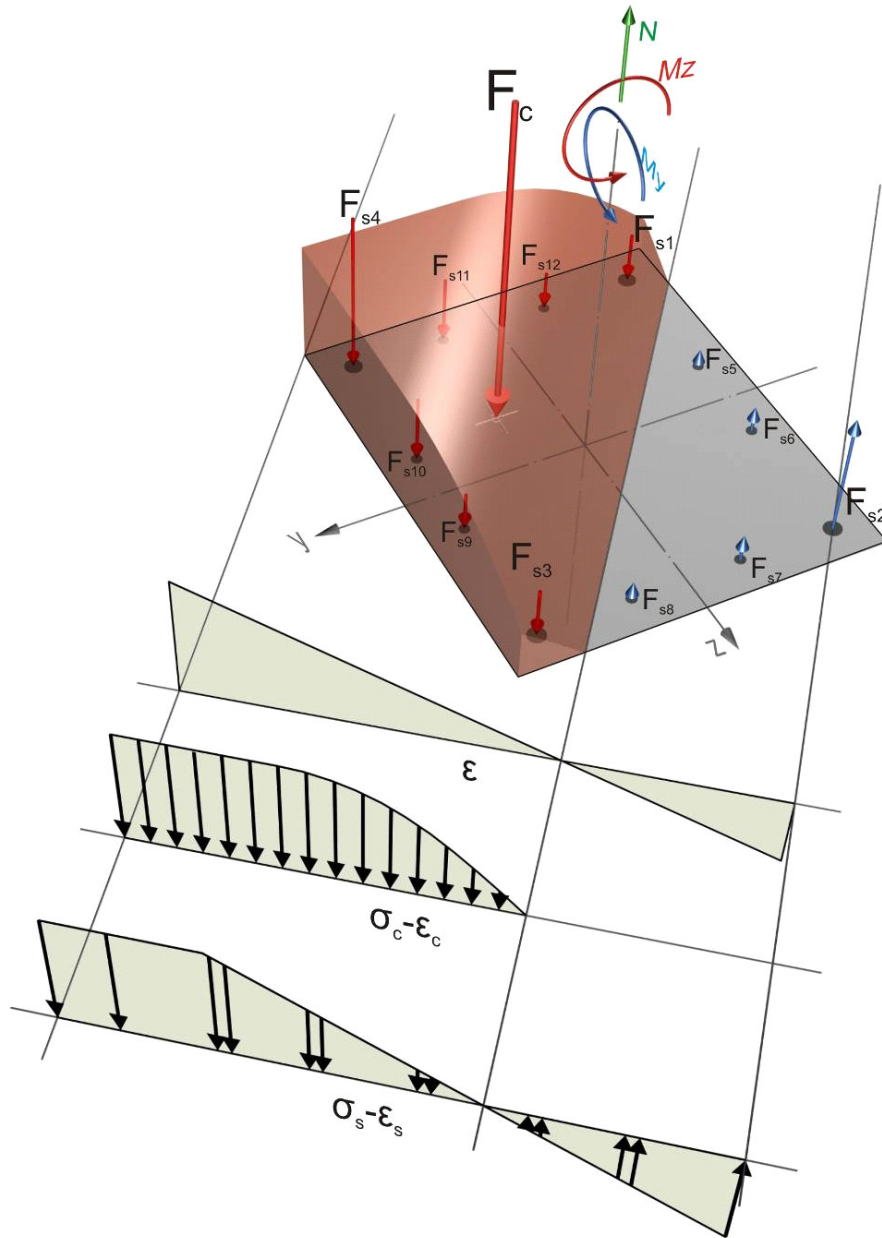
Next figure presents the cross-section of a seismically designed column with 3 cross-shaped shear reinforcement links enclosing longitudinal reinforcement bars at their corners. Strains are equal to zero along the neutral axis, and are positive or negative at any other section point.



Cross-section of a column under biaxial flexure

At the locations of reinforcing steel, stresses are tensile in the case of positive strains and compressive in the case of negative strains. Those stresses generate a force F_{si} at the center of each bar. Tensile stresses yield tensile forces which are symbolized by $\odot$, whereas compressive forces yield negative forces symbolized by $\ominus$.

In the region of the cross sectional area that is under tension there are no stresses developing in concrete, while in the region under compression exist only compressive stresses (both in concrete and reinforcement). Compressive stresses in concrete yield a compressive force F_c on concrete.



Internal forces (F_c, F_{si}) balance external loads (N, M_y, M_z)

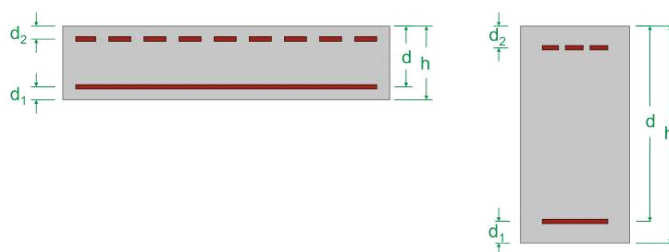
Cross sectional internal forces F_c and F_{si} cause loads N_{in} , M_{yin} , M_{zin} at the center of mass, where $N_{in} = F_c + \sum(F_{si})$, $M_{yin} = \sum z_c + \sum(F_{si} \cdot z_i)$ and

$$M_{zin} = F_c \cdot y_c + \sum (F_{si} \cdot y_i).$$

If external loads N , M_y , M_z are acting on the cross-section, then three equilibrium conditions must be satisfied: $N_{in}=N$, $M_{yin}=M_y$, $M_{zin}=M_z$.

The algorithms calculating the required reinforcement in a cross-section must take into account the distribution of reinforcement.

In the case of slabs and beams the location of reinforcing bars is known and it resides at distances d_1 and d_2 from the extreme upper and lower fibers of the cross-section.

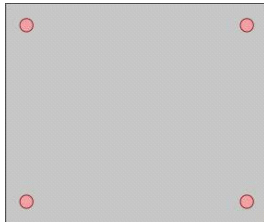


Reinforcement in slabs and beams is considered distributed

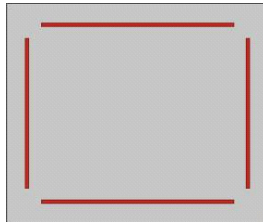
However, in the case of columns, distribution of reinforcement is not known at first. It depends on the reinforcement demand, the demand on crossties and other factors. On the above example of a column, it is not initially known whether the reinforcing bars that are going to be placed at the four corners of the member shall be of larger diameter (e.g. Ø25) than the bars that are going to be placed at the 8 internal corners of the crossties (e.g. Ø20).

For columns the two limit distributions of reinforcement are: the placement of reinforcement at the member corners and the placement of reinforcement evenly at all sides.

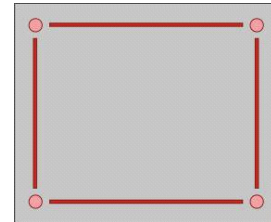
Placement of reinforcement in a column



(a) only at points
(dis=0.0)



(b) only distributed
(dis=1.0)



(c) mixed, at points and
distributed (0.0<dis<1.0)

Note: As shown in the application examples of column C1 on § 8.4, distribution (a) yields a reinforcement demand of $A_{s,cal}=3912 \text{ mm}^2$, distribution (b) for dis=1.0 yields $A_{s,cal}=5000 \text{ mm}^2$ and distribution (c) for dis=0.50 yields $A_{s,cal}=2188+2188=4376 \text{ mm}^2$.

2. Force undertaken by concrete

The value of the design compressive strength is defined as:

$$f_{cd} = \alpha_{cc} f_{ck} / \gamma_C$$

where:

γ_C is the partial safety factor for concrete (usually 1.50) and
 α_{cc} is the coefficient taking into account long term effects on the compressive strength and unfavorable effects resulting from the way the load is applied. The value of α_{cc} for use in a country should lie between 0.8 and 1.0 and may be found in its National Annex. The recommended value is 1.0.

e.g. for concrete strength class C30, $f_{cd}=1.0*30/1.50=20.0$ MPa

For the design of cross-sections, the following stress-strain relationship may be used, see figure below (compressive strain shown positive):

$$\sigma_c = f_{cd} \left[1 - \left(1 - \frac{\varepsilon_c}{\varepsilon_{c2}} \right)^n \right] \quad \text{for } 0 \leq \varepsilon_c \leq \varepsilon_{c2}$$

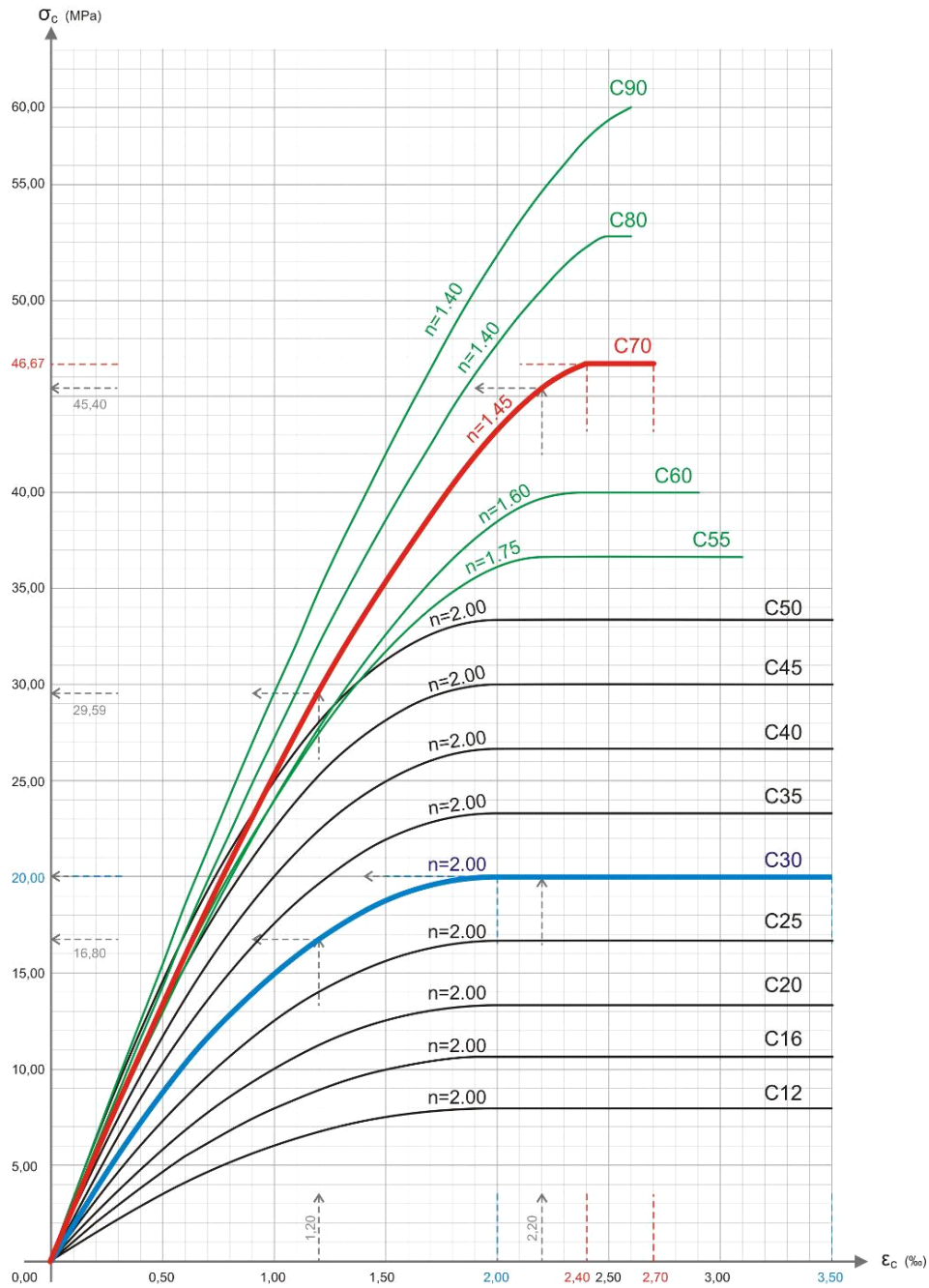
$$\sigma_c = f_{cd} \quad \text{for } \varepsilon_{c2} \leq \varepsilon_c \leq \varepsilon_{cu2}$$

where:

n is the exponent according to Table 3.1

ε_{c2} is the strain at reaching the maximum strength according to Table 3.1

ε_{cu2} is the ultimate strain according to Table 3.1



Stress-strain diagrams for each concrete strength class
for $a_{cc}=1.00$ and $\gamma_c=1.50$, where n is the exponent of each curve.

The stress-strain diagram of each concrete strength class comprises of two parts: a curved line and a horizontal line. For strength classes ≤ 50 MPa the curved line is parabolic ($n=2.0$), whereas for upper strength classes which are more brittle, $1.4 \leq n < 2.0$. For concrete strength classes ≤ 50 MPa ultimate strains are $\epsilon_{c2} = -2.0\text{‰}$ and $\epsilon_{cu2} = -3.5\text{‰}$, while for higher strength classes the two values tend to -2.6‰ and for C90 are equal to -2.6‰ , which means there is no plastic region.

Examples of C30 concrete ($\epsilon_{c2} = 2.0\text{‰}$, $\epsilon_{cu2} = 3.5\text{‰}$, $n = 2.0$):

- for $\epsilon_c = 1.2\text{‰}$
 $\sigma_c = 30 / 1.5 * [1 - (1 - 1.2/2.0)^2] = 20.0 * [1 - 0.16] = 20.0 * 0.84 = 16.80$ MPa
- for $\epsilon_c = 2.2\text{‰}$ $\sigma_c = 20.0$ MPa

Examples of C70 concrete ($\epsilon_{c2} = 2.4\text{‰}$, $\epsilon_{cu2} = 2.7\text{‰}$, $n = 1.45$):

- for $\epsilon_c = 1.2\text{‰}$
 $\sigma_c = 70 / 1.5 * [1 - (1 - 1.2/2.4)^{1.45}] = 46.67 * [1 - 0.366] = 46.67 * 0.634 = 29.59$ MPa
- for $\epsilon_c = 2.2\text{‰}$
 $\sigma_c = 70 / 1.5 * [1 - (1 - 2.2/2.4)^{1.45}] = 46.67 * [1 - 0.027] = 46.67 * 0.973 = 45.40$ MPa

The force undertaken by concrete is $F_c = \iint \sigma_c * d_y * d_z$ and its application point is

$$(y_c, z_c), \text{ where: } y_c = \frac{\left(\iint \sigma_c * d_y * d_z * z \right)}{F_c}, \quad z_c = \frac{\left(\iint \sigma_c * d_y * d_z * y \right)}{F_c}.$$

In the special case of a rectangular cross-section with width b subjected to uniaxial flexure with x the height of the compressive zone, the above integrals are specialized as shown below:

For $0 < \epsilon_c \leq \epsilon_{c2}$:

$$F_c = f_{cd} * b * x * \left(1 - \frac{1 - \left(1 - \frac{\epsilon_c}{\epsilon_{c2}} \right)^{n+1}}{(n+1) * \frac{\epsilon_c}{\epsilon_{c2}}} \right)$$

$$z_c = \frac{1}{2} * x * \frac{1 - 2 * \frac{\left(1 - \frac{\epsilon_c}{\epsilon_{c2}}\right)^{n+2} + (n+2) * \frac{\epsilon_c}{\epsilon_{c2}} - 1}{(n+1) * (n+2) * \left(\frac{\epsilon_c}{\epsilon_{c2}}\right)^2}}{1 - \frac{1 - \left(1 - \frac{\epsilon_c}{\epsilon_{c2}}\right)^{n+1}}{(n+1) * \frac{\epsilon_c}{\epsilon_{c2}}}}$$

For $\epsilon_{c2} \leq \epsilon_c \leq \epsilon_{cu2}$ ¹:

$$F_c = f_{cd} * b * x * \left(1 - \frac{1}{n+1} * \frac{\epsilon_{c2}}{\epsilon_c}\right)$$

$$z_c = \frac{1}{2} * x * \frac{\left(1 - \frac{\epsilon_{c2}}{\epsilon_c}\right)^2 + \frac{2 * (n+2) - (n+3) * \frac{\epsilon_{c2}}{\epsilon_c}}{(n+1)(n+2)} * n * \frac{\epsilon_{c2}}{\epsilon_c}}{1 - \frac{1}{n+1} * \frac{\epsilon_{c2}}{\epsilon_c}}$$

Examples of C30 concrete ($\epsilon_{c2}=2.0\%$, $n=2.0$), for $b=300$ mm $x=20$ mm:

for $\epsilon_c=1.2\%$ $F_c=576.00$ kN and $z_c=70.8$ mm

for $\epsilon_c=2.2\%$ $F_c=836.36$ kN and $z_c=76.3$ mm

Examples of C70 concrete ($\epsilon_{c2}=2.4\%$, $n=1.45$), for $b=300$ mm and $x=200$ mm:

for $\epsilon_c=1.2\%$ $F_c= 932.6$ kN and $z_c=68.2$ mm

for $\epsilon_c=2.2\%$ $F_c=1556.1$ kN and $z_c=70.4$ mm

Note:

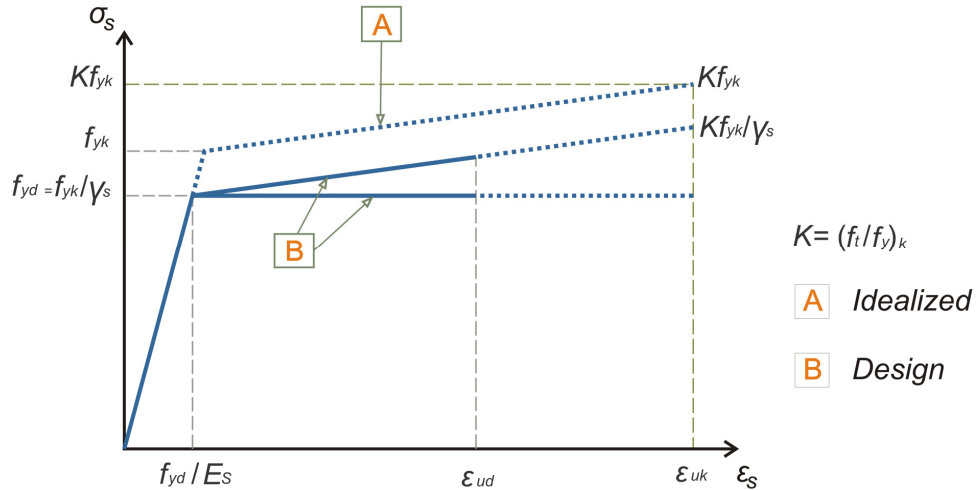
*The first worksheet “Concrete” of the file **StressesAndForces.xls**, which can be found in the installation directory of the supplementary software, can be used in order to easily check the previous and all other respective application*

¹ For the extreme case where $\epsilon_c = \epsilon_{c2}$ the two regions degenerate into common solution:

$$F_c = f_{cd} * b * x * \frac{n}{n+1}, \quad z_c = \frac{1}{2} * x * \frac{n+1}{n+2}$$

examples.

3. Force undertaken by reinforcing steel



*Design stress-strain diagram of reinforcing steel
(valid for tension and compression)*

If we symbolize the elastic strain limit of steel as ϵ_{yd} ($\epsilon_{yd} = f_{yd}/E_s$),

$$\sigma = \epsilon \cdot E_s \quad \text{for } 0 \leq \epsilon \leq \epsilon_{yd}$$

and

$$\sigma = f_{yd} + (k-1) \cdot f_{yd} \cdot (\epsilon - \epsilon_{yd}) / (\epsilon_{uk} - \epsilon_{yd}) \quad \text{for } \epsilon_{yd} < \epsilon \leq \epsilon_{ud}, \text{ where } \epsilon \geq 0$$

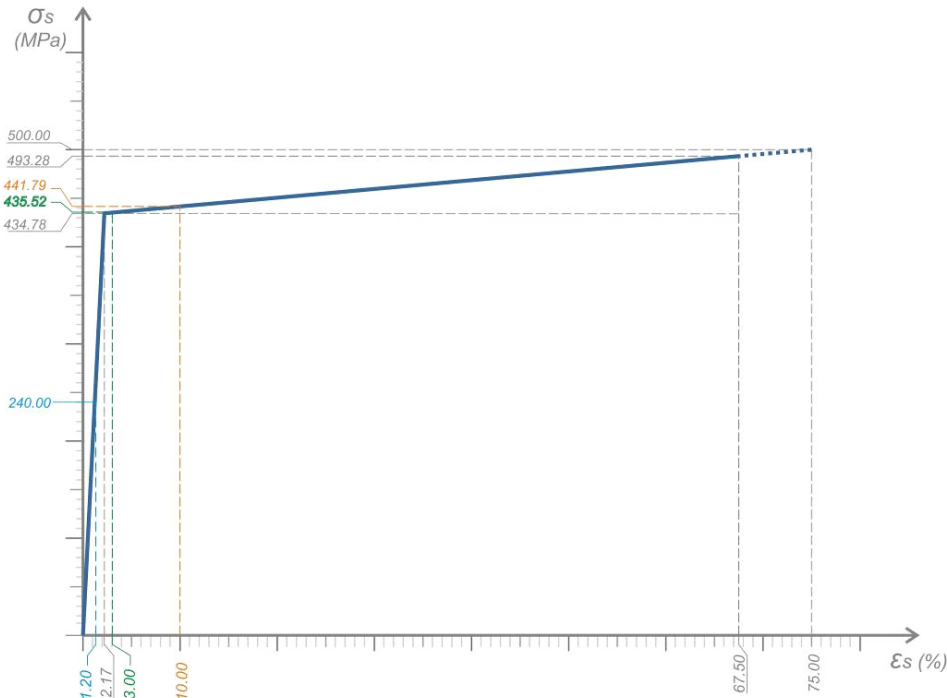
Typical values of the parameters for B500c steel are:

$$\epsilon_{uk} = 75\text{‰}, \epsilon_{ud} = 0.9 \cdot \epsilon_{uk} = 67.5\text{‰} \text{ and } k = f_t/f_{yk} = 1.15.$$

$$\text{In addition, for } \gamma_s = 1.15, f_{yd} = 500/1.15 = 434.78 \text{ MPa}$$

$$\text{and because } E_s = 200 \text{ GPa, } \epsilon_{yd} = f_{yd}/E_s = 434.78/(200 \cdot 10^3) = 2.17\text{‰}.$$

Also $k \cdot f_{yk}/\gamma_s = 1.15 \cdot 500/1.15 = 500 \text{ MPa}$ (the fact that both k and γ_s are equal to 1.15 is coincidental), thus the stress-strain diagram for this specific steel quality is the following:



Design stress-strain diagram of B500c reinforcing steel appropriate for use in seismically designed structures (valid for tension and compression).

Examples for B500c reinforcing steel:

- for $\epsilon_s = 1.20\%$ $\sigma_s = (1.20/1000) \cdot 200 \cdot 10^3 = 240 \text{ MPa}$,
- for $\epsilon_s = 3.00\%$ $\sigma_s = 434.78 + (1.15 - 1) \cdot 434.78 \cdot (3.00 - 2.17) / (75.00 - 2.17) = 434.78 + 0.74 = 435.52 \text{ MPa}$
- for $\epsilon_s = 10.00\%$ $\sigma_s = 434.78 + (1.15 - 1) \cdot 434.78 \cdot (10.00 - 2.17) / (75.00 - 2.17) = 434.78 + 7.01 = 441.79 \text{ MPa}$
- for $\epsilon_s = 67.5\%$ $\sigma_s = 434.78 + (1.15 - 1) \cdot 434.78 \cdot (67.5 - 2.17) / (75.00 - 2.17) = 434.78 + 58.50 = 493.28 \text{ MPa}$

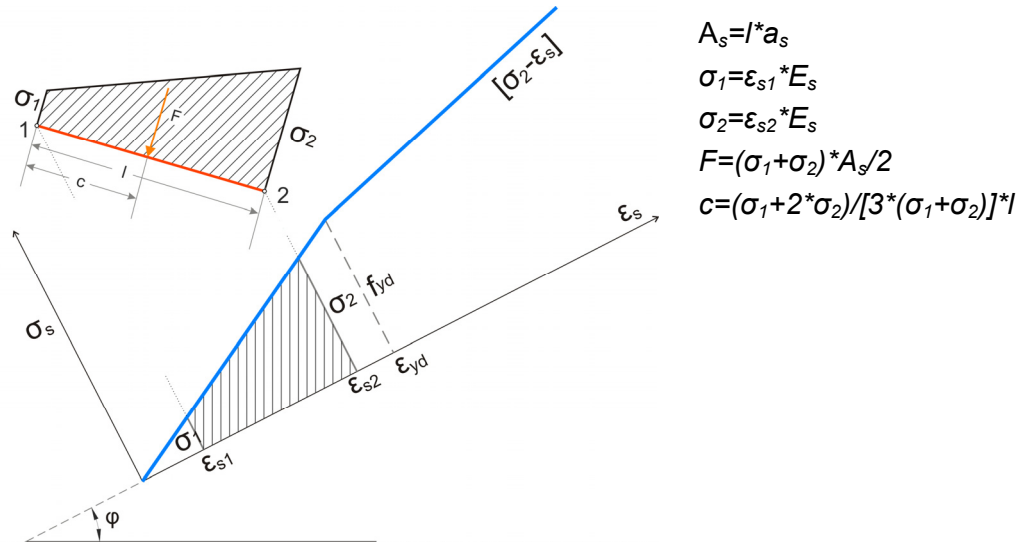
A $\varnothing 20$ reinforcing bar ($A_s = 314 \text{ mm}^2$) for $\epsilon_s = 1.20\%$ yields force

$$F_s = 314 \text{ mm}^2 \cdot 240 / 1000 \text{ MPa} = 75.36 \text{ kN}$$

A distributed reinforcement of 314 mm^2 on a length $l = 300 \text{ mm}$ ($a_s =$

=314/300/1000=1047 mm²/m), depending on the deformations at the two ends 1 and 2, yields the following forces:

1st case, $\epsilon_{s2} \leq \epsilon_{yd}$:



Example application of 1st case:

$l=300$ mm, $a_s=1047$ mm²/m, $\epsilon_{s1}=0.5\text{‰}$ and $\epsilon_{s2}=1.5\text{‰}$

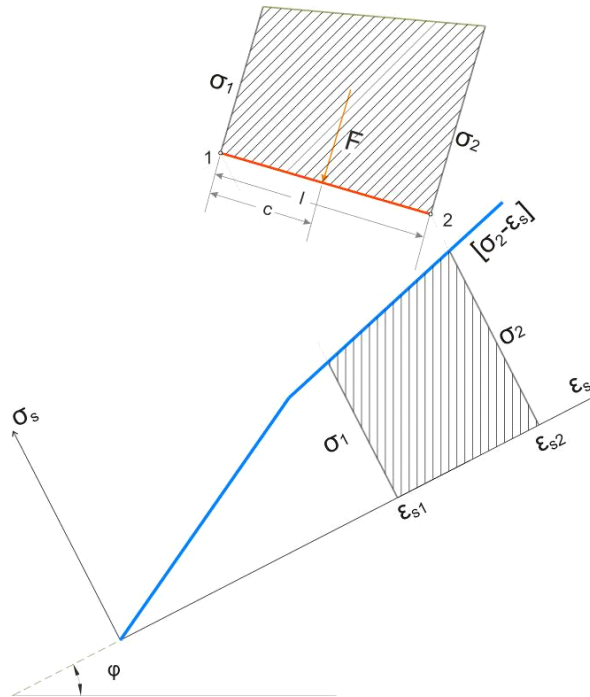
It is $A_s=314$ mm², $\sigma_1=(0.5 \cdot 10^{-3}) \cdot 200 \cdot 10^3$ MPa=100 MPa and

$\sigma_2=(1.5 \cdot 10^{-3}) \cdot 200 \cdot 10^3$ MPa=300 MPa,

$F_s=[(100+300) \cdot 10^3 \text{ kPa} \cdot 314 \cdot 10^{-6} \text{ mm}^2] / 2 = 62.8$ kN,

$c=(0.1+2 \cdot 0.3) / [3 \cdot (0.1+0.3)] \cdot 300 = 175$ mm

2nd case, $\varepsilon_{yd} \leq \varepsilon_2$:



$$A_s = l \cdot a_s$$

$$\sigma_1 = f_{yd} + (k-1) \cdot f_{yd} \cdot (\varepsilon_{s1} - \varepsilon_{yd}) / (\varepsilon_{uk} - \varepsilon_{yd})$$

$$\sigma_2 = f_{yd} + (k-1) \cdot f_{yd} \cdot (\varepsilon_{s2} - \varepsilon_{yd}) / (\varepsilon_{uk} - \varepsilon_{yd})$$

$$F = (\sigma_1 + \sigma_2) \cdot A_s / 2$$

$$c = (\sigma_1 + 2 \cdot \sigma_2) / [3 \cdot (\sigma_1 + \sigma_2)] \cdot l$$

Example application of 2nd case:

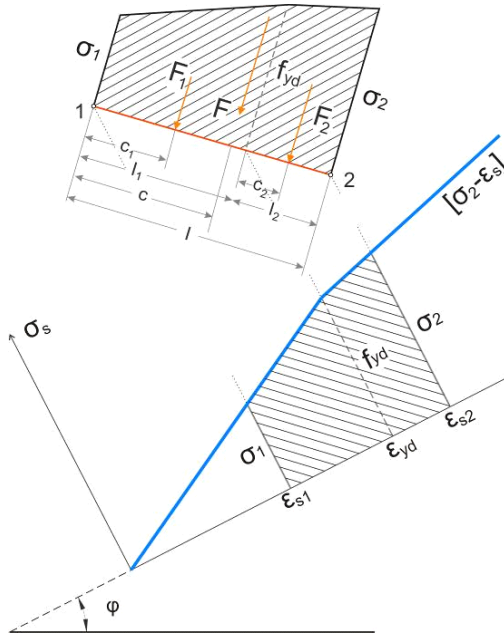
For $\varepsilon_{s1} = 3.0\text{‰}$ and $\varepsilon_{s2} = 10.0\text{‰}$

$\sigma_1 = 435.52 \text{ MPa}$, $\sigma_2 = 441.79 \text{ MPa}$,

$F = (435.52 + 441.79) \cdot 10^3 \text{ kPa} \cdot 314 \cdot 10^{-6} \text{ mm}^2 / 2 = 137.7 \text{ kN}$

$c = (435.52 + 2 \cdot 441.79) / [3 \cdot (435.52 + 441.79)] \cdot 309 = 150 \text{ mm}$

3rd case, $\varepsilon_1 \leq \varepsilon_{yd} \leq \varepsilon_2$



$$\begin{aligned}
 A_s &= l \cdot a_s \\
 l &= l_1 + l_2 \\
 l_1 &= l \cdot (\varepsilon_{yd} - \varepsilon_1) / (\varepsilon_2 - \varepsilon_1) \\
 A_{s1} &= l_1 \cdot a_s \\
 \sigma_1 &= \varepsilon_{s1} \cdot E_s \\
 F_1 &= (\sigma_1 + f_{yd}) \cdot A_{s1} / 2 \\
 c_1 &= (\sigma_1 + 2 \cdot f_{yd}) / [3 \cdot (\sigma_1 + f_{yd})] \cdot l_2 = l - l_1 \\
 A_{s2} &= l_2 \cdot a_s \\
 \sigma_2 &= f_{yd} + (k-1) \cdot f_{yd} \cdot (\varepsilon_{s2} - \varepsilon_{yd}) / (\varepsilon_{uk} - \varepsilon_{yd}) \\
 F_2 &= (f_{yd} + \sigma_2) \cdot A_{s2} / 2 \\
 c_2 &= (f_{yd} + 2 \cdot \sigma_2) / [3 \cdot (f_{yd} + \sigma_2)] \cdot l_2 \\
 F &= F_1 + F_2 \\
 c &= [F_1 \cdot c_1 + F_2 \cdot (l_1 + c_2)] / (F_1 + F_2)
 \end{aligned}$$

Example application of 3rd case:

For $\varepsilon_{s1} = 1.5\text{‰}$ and $\varepsilon_{s2} = 3.0\text{‰}$

$$l_1 = 300 \cdot (2.17 - 1.5) / (3 - 1.5) = 300 \cdot 0.449 = 135 \text{ mm}, A_{s1} = 314 \cdot 0.449 = 141 \text{ mm}^2,$$

$$l_2 = 165 \text{ mm}, A_{s2} = 173 \text{ mm}^2$$

$$\sigma_1 = 300.00 \text{ MPa}, f_{yd} = 434.78 \text{ MPa}, \sigma_2 = 435.52 \text{ MPa},$$

$$F_1 = (300.00 + 434.78) \cdot 10^3 \text{ kPa} \cdot 140 \cdot 10^{-6} \text{ mm}^2 / 2 = 518.02 \text{ kN}$$

$$c_1 = (300.00 + 2 \cdot 434.78) / [3 \cdot (300.00 + 434.78)] \cdot 135 = 72 \text{ mm}$$

$$F_2 = (434.78 + 435.52) \cdot 10^3 \text{ kPa} \cdot 173 \cdot 10^{-6} \text{ mm}^2 / 2 = 752.81 \text{ kN}$$

$$c_2 = (434.78 + 2 \cdot 435.52) / [3 \cdot (434.78 + 435.52)] \cdot 165 = 83 \text{ mm}$$

$$F = F_1 + F_2 = 518.02 + 752.81 = 1270.83 \text{ kN}$$

$$c = [518.02 \cdot 72 + 752.81 \cdot (135 + 83)] / 1270.83 = 159 \text{ mm}$$

Note:

The second worksheet "Steel" of the file **StressesAndForces.xls**, which can be found in the installation directory of the accompanying software, can be used in order to easily check the previous and all other respective application examples. Mainly it allows us to confirm the accuracy of the design calculations made with the help of Pi-Design software.

4. Design of a cross-section

Generally we have two design problems:

Problem A

For given external design loads (N_d , M_{dy} , M_{dz}) applied on the center of mass of the cross-section, the calculation of the reinforcement demand $A_{s,cal}$ is requested.

This problem is solved by determining the combination of the required reinforcement $A_{s,cal}$ and a specific location of the neutral axis that yields $N_d=N_{in}$, $M_{dy}=M_{yin}$, $M_{dz}=M_{zin}$.

Problem B

For given reinforcement $A_{s,eff}$ placed in the cross-section, compressive force N_d and direction dir of neutral axis in degrees, the calculation of the resisting moment M_{Rd} towards direction dir is requested. Alternatively, instead of angle dir , fields dM_y and dM_z which constitute the two components of the direction of the neutral axis, i.e. $dir=atan(dM_z/dM_y)$, can be filled.

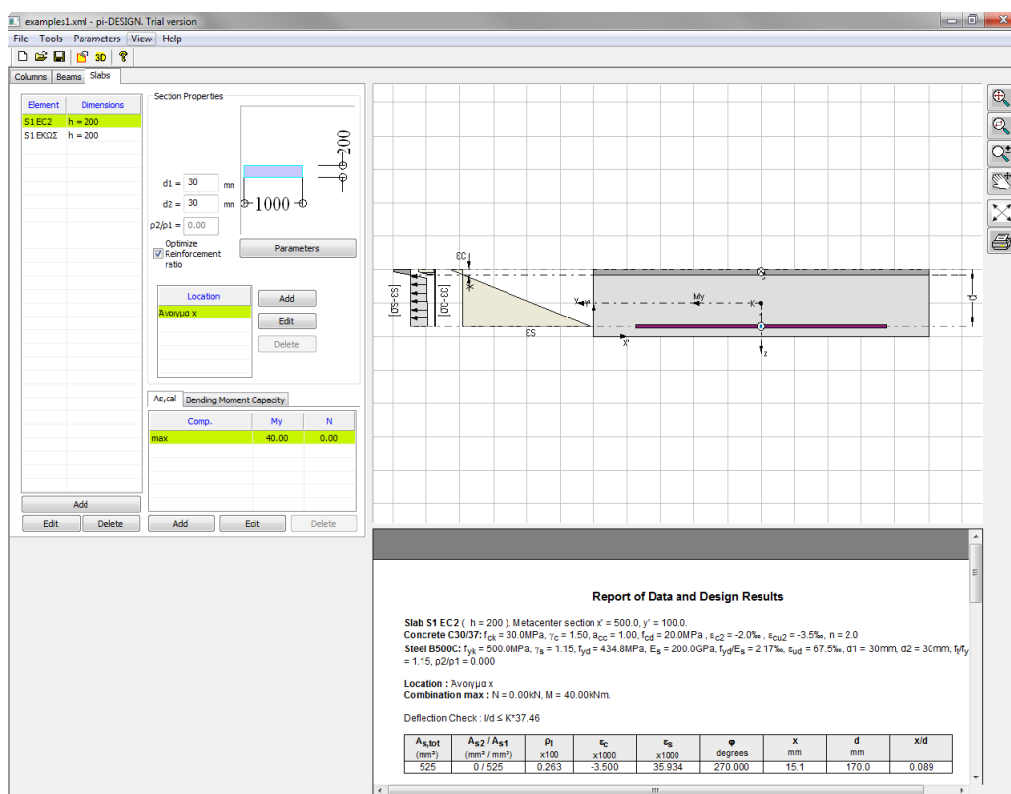
This problem is solved by specifying the neutral axis that yields $N_{in}=N_d$ and moments M_{yin}, M_{zin} that combined produce a moment towards direction dir . Then the solution is $M_{Rd} = \sqrt{M_{yin}^2 + M_{zin}^2}$.

Example applications of solutions to problems A and B with the use of pi-DESIGN

All example applications for the design of slab, beam and column can be found in file examples1.xml.

Slab 'S1 EC2'²

Slab thickness is 200 mm, materials C30/B500c, $d_1=30$ mm and design moment $M_d=40.0$ kNm.



Screenshot of the program with input and result data³ for slab 'S1 EC2'

The result is only tension reinforcement $A_{s1}=525$ mm² [$\rho_1=0.263\%$, $\epsilon_c=-3.50\text{‰}$, $\epsilon_s=35.93\text{‰}$, $x/d=0.089$]. The solution details are available on-screen and in printed form on the input/output data report generated by the program⁴.

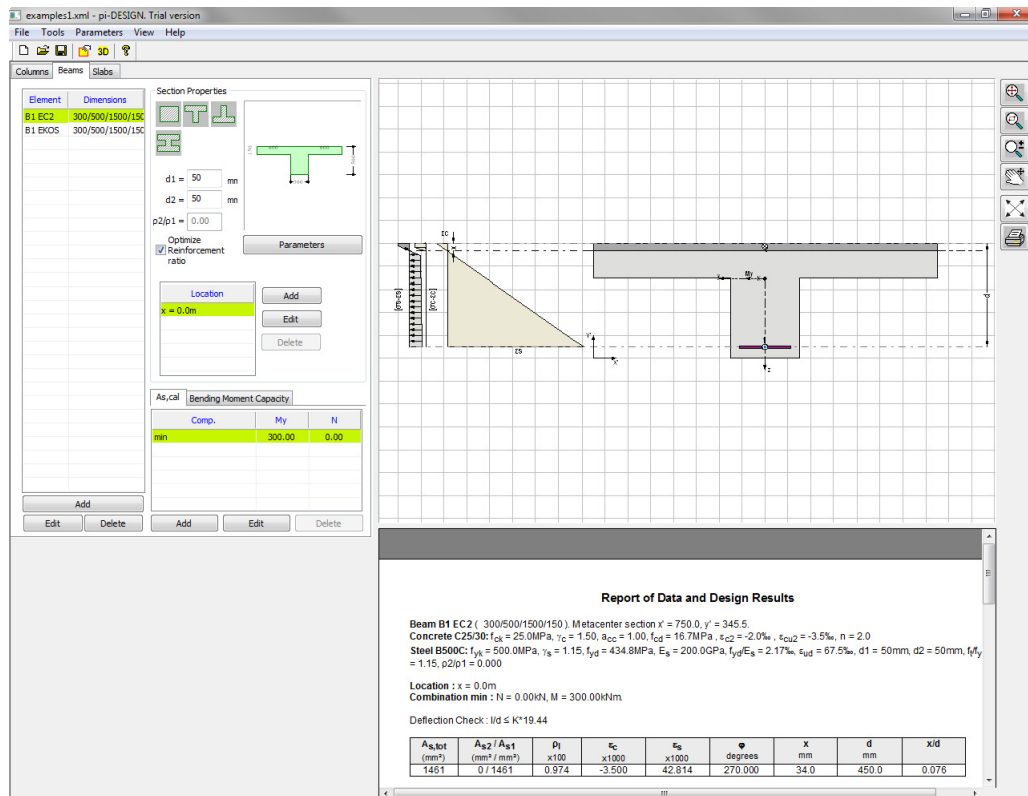
² The index EC2 denotes that slab S1 is designed according to Eurocode 2.

³ Analytical input and result data can be utilized to check the accuracy of the solution.

⁴ Slab 'S1 EKOS' of the same example application, by keeping the same input data and choosing the former Greek Reinforcement Concrete Code (EKOS), yields $A_{s1} = 571$ mm² [$\rho_1=0.285\%$, $\epsilon_c=-1.66\text{‰}$, $\epsilon_s=10.0\text{‰}$, $x/d=0.143$].

Beam 'B1 EC2'

The T-beam has a web with dimensions 300/500 and effective width of flange 1500/150, materials C25/B500c, $d_1=50$ mm and design moment $M_d=300.0$ kNm.



Screenshot of the program with input and result data for beam "B1 EC2"

The result is only tension reinforcement $A_{s1}=1461$ mm² [$\rho_i=0.974\%$ ⁵, $\epsilon_c=-3.50\%$, $\epsilon_s=42.81\%$, $x/d=0.076$]⁶.

In order for the reinforcement demand to be satisfied, 5Ø20 (1570 mm²)

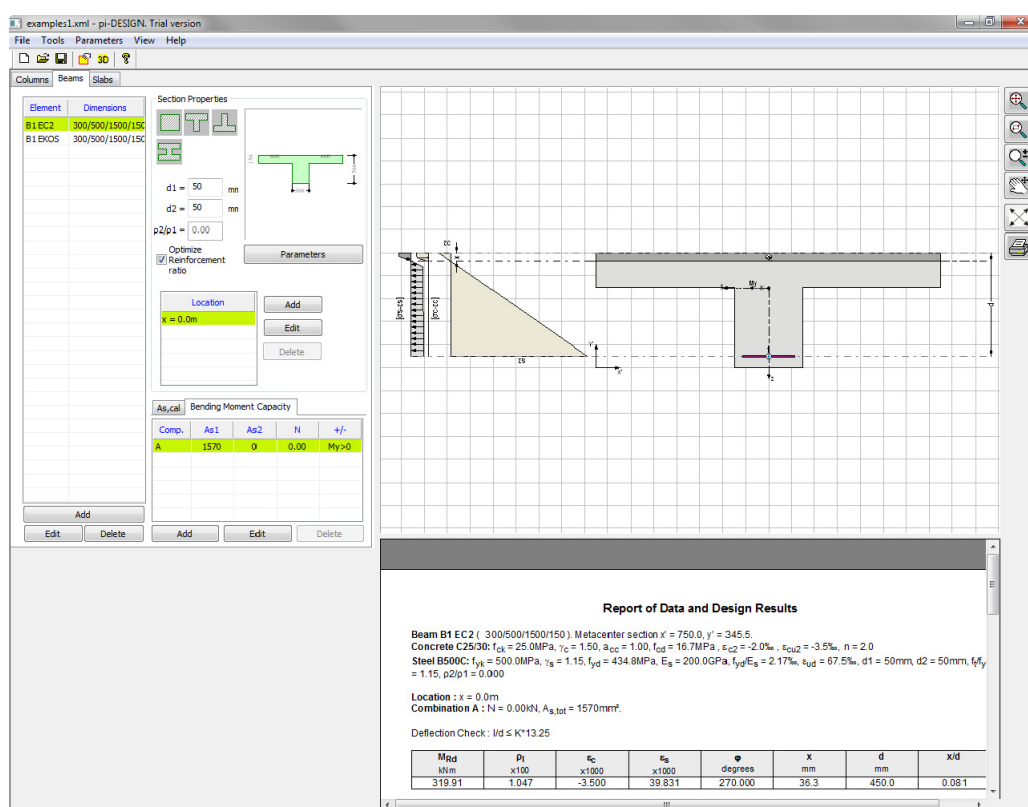
⁵ The percentage of reinforcement in T-beams is normalized in terms of the web area and not the total section area.

⁶ Beam 'B1 EKOS' for the same input data but with application of the former Greek R/C Code yields $A_{s1}=1609$ mm² [$\rho_i=1.073\%$, $\epsilon_c=-1.50\%$, $\epsilon_s=10.0\%$, $x/d=0.130$].

(table 1) can be placed. This amount of reinforcement is larger than the required, thus the capacity moment of the cross-section is greater. The latter is called resisting moment M_{Rd} and its value is calculated by the program by applying the reverse procedure, as presented in application example 'B2 EC2'.

Beam 'B1 EC2' calculation of resisting moment

The T-beam has a web with dimensions 300/500 and effective width of flange 1500/150, materials are C25/B500c, $d_1=50$ mm and reinforcement $A_s=1570$ mm². Determination of the positive resisting moment is requested.

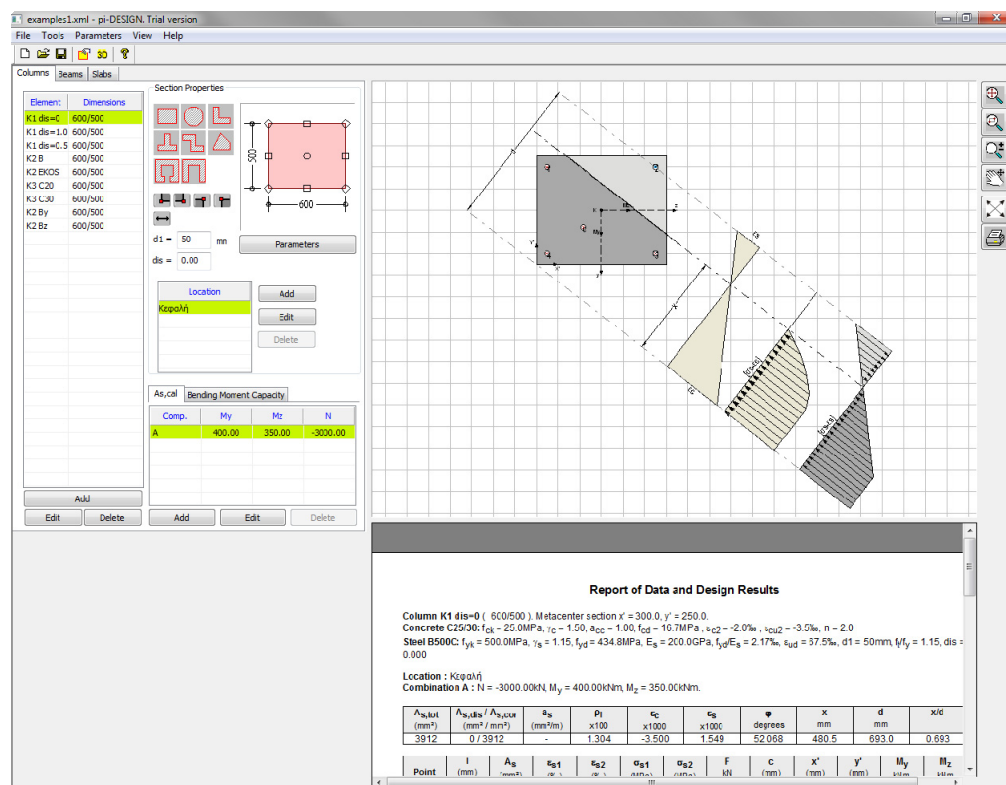


Screenshot of the program with input and result data for beam "B1 EC2"

The calculated resisting moment is $M_{Rd}=319.91$ kNm [$\epsilon_c=-3.50\text{‰}$, $\epsilon_s=39.83\text{‰}$, $x/d=0.081$].

Column 'C1 dis=0'

The column with dimensions 600/500 is the column of the first application example of this chapter without any distributed⁷ reinforcement (dis=0.0), materials C25/B500c, $d_1=50$ mm and design loads $M_{yd}=400$ kNm, $M_{zd}=350$ kNm, $N_d=-3000$ kN.

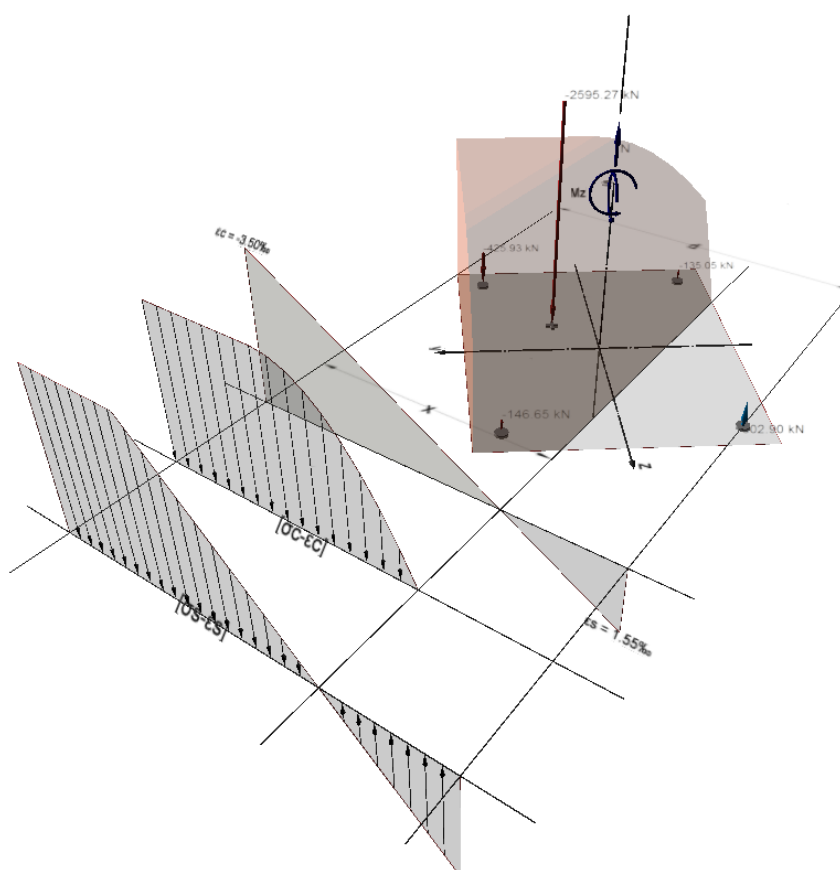


Screenshot of the program with input and result⁸ data and the 2D representation of the solution for columns 'C1 dis=0'

⁷ Reinforcement placed only in the corners (dis=0.0) of seismically designed columns is a rare case. Practically that could be the case only for 300/300 columns as referred in Volume A', §3.1.4. The assumption of placing 50% of the reinforcement in the corners and 50% at the sides (dis=0.50) would be more realistic, while the assumption of placing 100% of the reinforcement at the sides (dis=1.0) is always conservative and yields higher reinforcement demands.

⁸ Internal forces F_c , F_{si} and the coordinates of their application point expressed on the section's $x'y'$ local coordinate system, enable us to check the validity of the solution.

The calculated reinforcement demand at the corners of the column is $A_{s,cor}=3912 \text{ mm}^2$ [$\rho_l=1.304\%$, $\varepsilon_c=-3.50\%$, $\varepsilon_s=1.55\%$, $\phi=52.07^\circ$, $x=480 \text{ mm}$]⁹.

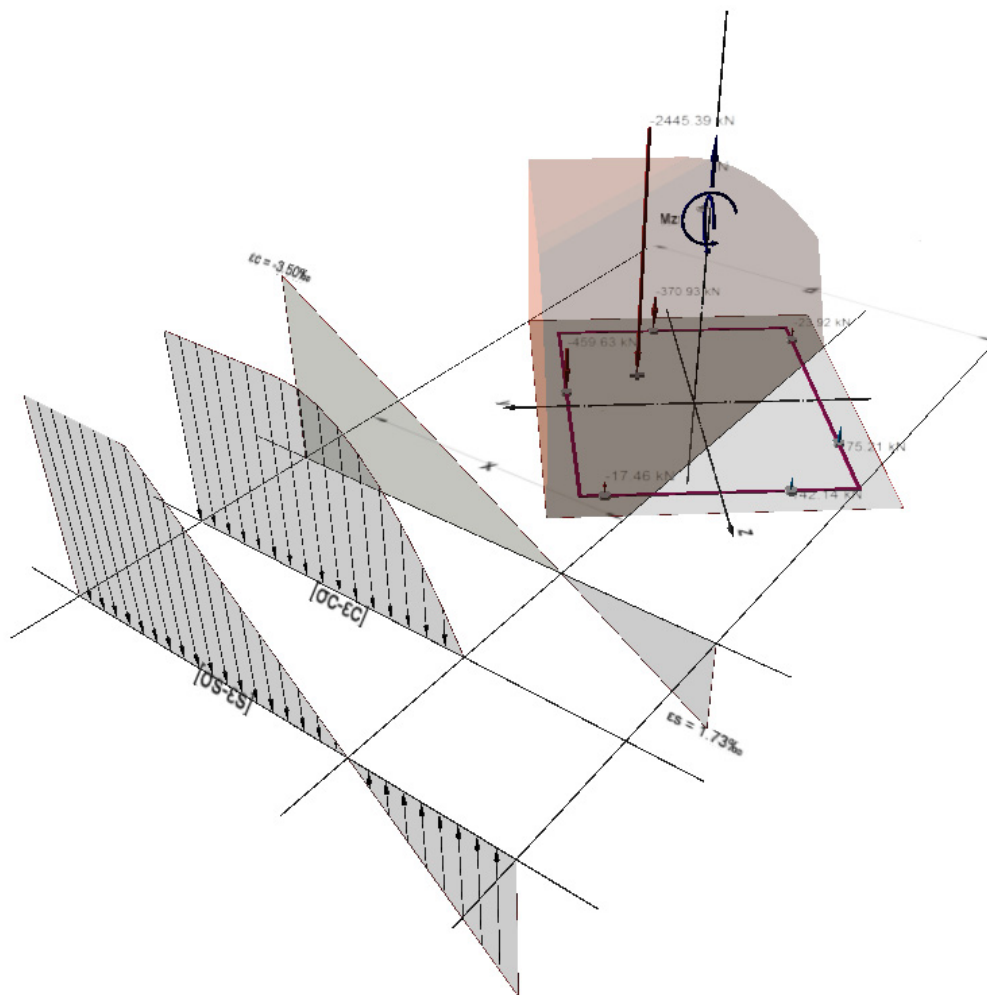


*Screenshot of the program featuring a 3D representation of the solution for column 'C1 dis=0.0'*¹⁰

⁹ Angle ϕ and length x define mathematically the location of the neutral axis determined by software algorithms.

¹⁰ 3D representation enables comprehension of the design and validation of the solution. It is available to any design case through the '3D' option.

Results indicate that the total required reinforcement is $A_{s,dis}=5000 \text{ mm}^2$ [$\rho_l=1.667\%$, $\epsilon_c=-3.50\%$, $\epsilon_s=1.73\%$, $\theta=50.95^\circ$, $x=466 \text{ mm}$].



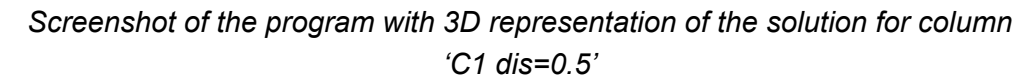
Screenshot of the program featuring 3D representation of the solution for column 'C1 dis=1.0'

Column 'C1 dis=0.5' ¹¹

Same example as above with the only difference lying in the fact that 50% of the reinforcement is distributed (dis=0.50) and the rest 50% is placed in the corners.

Results indicate corner reinforcement demand $A_{s,cor}=2188 \text{ mm}^2$ and distributed reinforcement demand $A_{s,dis}=2188 \text{ mm}^2$, thus total reinforcement demand $A_{s,tot}=2*2188=4376 \text{ mm}^2$ [$\rho_l=1.459\%$, $\epsilon_c=-3.50\%$, $\epsilon_s=1.63\%$, $\phi=51.60^\circ$, $x=474 \text{ mm}$]

¹¹ Maintaining the input data of column 'C1 dis0.5' and choosing the former Greek R/C Code (in our examples 'K2 EKOS' element), we get $A_{s,tot}=2*2781=5562 \text{ mm}^2$ [$\rho_l=1.854\%$, $\epsilon_c=-3.50\%$, $\epsilon_s=1.44\%$, $\phi=51.71^\circ$, $x=492 \text{ mm}$]. The difference between the reinforcement demand between EKOS (5562 mm^2) and EC2 (4380 mm^2) is due to factor α_{cc} that multiplies the compressive strength of concrete (see §8.2) and is equal to 0.85 in EKOS whereas its value in EC2 is 1.00.



Column 'C2 B'

The column of this application example is similar to the previous ones, dealing though with problem 'B' for $A_s=4476 \text{ mm}^2$ and $(dM_y=400, dM_z=350)$ or equivalently angle $\text{dir}=41.19^\circ$.

The calculated resisting moment is $M_{Rd}=537.39 \text{ kNm}$ [$\epsilon_c=-3.50\text{‰}$, $\epsilon_s=1.64\text{‰}$, $\varnothing=51.606^\circ$, $x=473 \text{ mm}$] in direction $\text{dir}=41.19^\circ$. This moment is analyzed in moments $M_y=404.4 \text{ kNm}$ and $M_z=353.9 \text{ kNm}$.

Column 'C2 By'

The column of this application example is similar to the previous ones and is treated as a 'B' problem for given reinforcement $A_s=4476 \text{ mm}^2$ and $(dM_y=1, dM_z=0)$ or angle $\text{dir}=0^\circ$.

The calculated resisting moment toward y axis is $M_{Rd}=701.75 \text{ kNm}$ [$\epsilon_c=-3.50\text{‰}$, $\epsilon_s=1.49\text{‰}$, $\varnothing=0.0^\circ$, $x=386 \text{ mm}$].

Column 'C2 Bz'

The column of this application example is similar to the previous ones and is treated as a 'B' problem for given reinforcement $A_s=4476 \text{ mm}^2$ and $(dM_y=0, dM_z=1)$ or angle $\text{dir}=90^\circ$.

The calculated resisting moment toward z axis is $M_{Rd}=579.29 \text{ kNm}$ [$\epsilon_c=-3.50\text{‰}$, $\epsilon_s=1.41\text{‰}$, $\varnothing=90.00^\circ$, $x=320.5 \text{ mm}$].

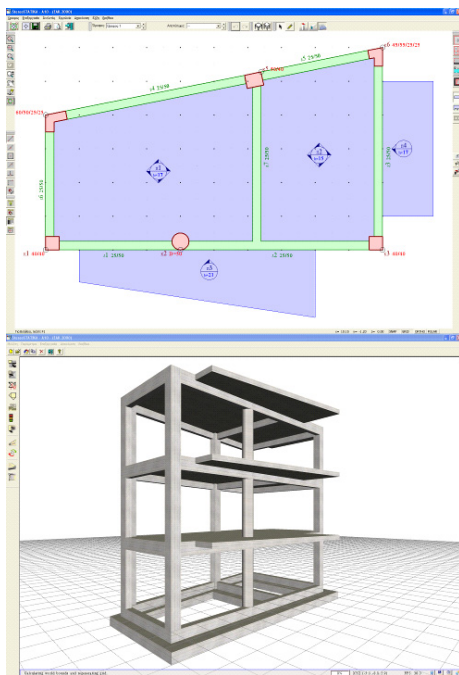
Note:

In each actual building study there are floors and structural members (columns, beams, slabs).

Each structural member has characteristic cross-sections, i.e. locations in

which design reinforcement is calculated. In the simplest case a column's characteristic cross-sections are its head and its base while the characteristic cross-sections of a beam are its two supports and the span. In case drawings of reinforcement details are required, design calculations are performed for cross-sections at constant distances of 100mm.

For each cross-section numerous load combinations exist. If these combinations yield a maximum and a minimum stress like in the case of beams where the dominant stress is uniaxial, then the combinations become two. However, in the case of columns, since the loading is biaxial, by default the most unfavorable combination (the combination that yields the heaviest reinforcement demand) is not known, thus all 33 or more loading combinations must be examined.



The small 3-floored building studied includes 6 columns, 7 beams and 4 slabs at each slab, while its base includes 7 foundation beams and 7 foundation footings.

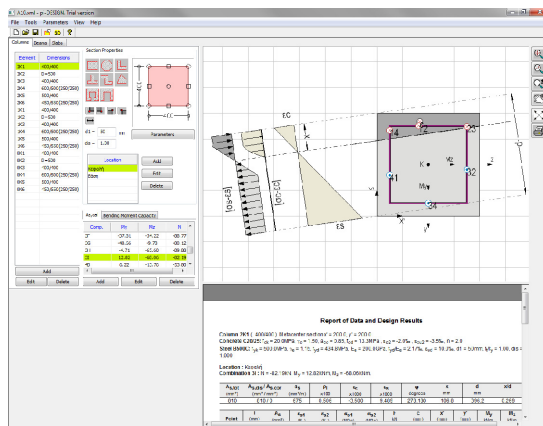
Taking into account the above data, the design calculations required are:

- For columns: 2 cross-sections $\times$ 33 load combinations per section = $7 \times 2 \times 33 = 462$ biaxial bending design calculations.
- For beams, 150 sections $\times$ 2 load combinations per section = 300 uniaxial bending design calculations.
- For slabs: 4 span cross-sections per slab and 2 cross-sections per slab at the supports between consecutive slabs = $4 \times 2 + 2 \times 4 = 16$ design calculations.
- For the foundation: 150 cross-sections $\times$ 2 load combinations for the webs of T-shaped foundations and 2 load combinations per footing $\times$ 7 footings = $150 \times 2 + 2 \times 7 = 314$ uniaxial bending design calculations.

For the entire small 3-floored building a total of $3 \times 462 = 1386$ biaxial and $3 \times 300 + 16 + 314 = 1230$ uniaxial bending design calculations are required!

File A10.xml that is included in the supplementary version of Pi-Design contains all the above members, cross-sections and combinations.

For a larger building the design calculations file includes tenths of thousands of elements and is almost impossible to be created manually, while can be very easily created by an automated analysis software such as StereoSTATIKA. File A10.xml has been created automatically by StereoSTATIKA, project A10.



In addition, the supplementary version of Pi-Design includes file A10.rar which contains the whole project from StereoSTATIKA and can be viewed and edited in any version (professional, educational or trial) of StereoSTATIKA 5.0 or newer. Project A10 must first be saved and then chosen. After that, 'design' must be chosen from the

central options and then 'analytical design'. File A10.xlm is automatically created and Pi-Design application is run including all designed cross-sections with full input/output data. Since locating the most unfavorable loading combination for each single cross-section is extremely difficult for the user, there is the 'Calc Max As' command under 'Tools' pull down menu.

5. Cost-effectiveness of materials

For seismically resistant structures in Greece reinforcing steel is of ductility category s, while its strength capacity varies from 400 MPa to 600 MPa. For reasons related to steel industry the strength capacity class used is 500 MPa, thus the only available reinforcing steel type is B500s.

For projects developed in coastal areas, thus susceptible to corrosion of reinforcing steel, concrete with cement concentration of at least 330 kg/m³ is required (see Volume A', § 2.4.1). This requirement leads to a concrete strength class of C25, or even better C30, if not higher.

The high strength capacity of concrete requires less reinforcement mainly for columns. The reduction in reinforcement demand is less for beams and close to none for slabs.

Let's discuss application example 'C3 C30' which is similar to 'C1 dis=0.5' with the only difference lying in the fact that concrete of strength class C30 instead of C25 is used.

Design calculation results yield a total reinforcement demand of $A_{s,tot}=2 \times 1498.5=2997 \text{ mm}^2$ [$\rho_l=0.999\%$, $\varepsilon_c=-3.50\text{‰}$, $\varepsilon_s=1.906\text{‰}$, $\varnothing=51.45^\circ$, $x=449.7 \text{ mm}$]. The above reinforcement is 30% less than the one calculated for C25 concrete [4376 mm^2 (1.46%)].

If we consider an average column height of 3000 mm and a reinforcing bar anchorage length equal to 1000mm for C25 concrete, the volume of the required reinforcement is $V=4360 \times 10^{-6} \times 4000/1000=0.01744 \text{ m}^3$. For mass per unit volume of steel equal to 7850 kg/m^3 , the mass of reinforcing steel is 137 kg. For C30 concrete the required reinforcing steel mass is 94kg.

The quantity of concrete is the same in both cases: $V=0.5 \times 0.60 \times 3.0=0.9 \text{ m}^3$

The difference between the price of C30 and C25 concrete is about 4 €/m^3 , thus the cost of C30 concrete is $0.9 \text{ m}^3 \times 4.0 \text{ €/m}^3=3.60 \text{ €}$. The difference in reinforcing steel quantity is $137-94=43 \text{ kg}$ and for an indicative price of about 0.5 €/kg the difference is 21.5 € compared to 3.6 € .

The difference in cost is clearly in favor of C30 concrete regardless of the number of additional cost factors taken into account.

So, by using C30 concrete instead of C25 we would save 17.9 € for the specific column.

On the other hand, if we used concrete of strength class C20 (application example 'C3 C20', the required reinforcement would be $A_{s,tot}=2 \times 2989=5978 \text{ mm}^2$ [$\rho_l=1.993\%$, $\varepsilon_c=-3.50\text{‰}$, $\varepsilon_s=1.378\text{‰}$, $\varnothing=51.755^\circ$, $x=497.9 \text{ mm}$], and the cost would be higher.

In many cases the placed reinforcement is the minimum required by the Design Code, e.g. equal to 1% of the cross-sectional area in the case of columns. In that case there might be no reduction in cost, but there is considerable increase in the member's capacity which could be extremely useful in the case of a strong earthquake, during which the member responds inelastically.

6. Tables

TABLE 1

Ø mm	Mass kg/m	Area of reinforcement in mm ²								
		1 pcs	2 pcs	3 pcs	4 pcs	5 pcs	6 pcs	7 pcs	8 pcs	9 pcs
5	0.154	20	39	59	78	98	118	137	157	176
6	0.222	28	57	85	113	142	170	198	226	255
8	0.395	50	101	151	201	252	302	352	402	453
10	0.617	79	157	236	314	393	471	550	628	707
12	0.888	113	226	339	452	565	678	791	904	1017
14	1.210	154	308	462	616	770	924	1078	1232	1386
16	1.580	201	402	603	804	1005	1206	1407	1608	1809
18	2.000	254	508	762	1016	1270	1524	1778	2032	2286
20	2.470	314	628	942	1256	1570	1884	2198	2512	2826
25	3.850	491	982	1473	1964	2455	2946	3437	3928	4419
28	4.833	616	1232	1847	2463	3079	3695	4310	4926	5542
32	6.310	804	1608	2412	3216	4040	4824	5628	6432	7236

Example: 4Ø14 corresponds to reinforcement area of 616 mm² and to mass 4*1.21=4.84 kg

Example: 2Ø20 corresponds to reinforcement area of 628 mm² and to mass 2*2.47=4.94 kg

TABLE 2

Lightweight structural reinforcement grids - B500A - ELOT 1421-2

Dimensions of sheet	Type of grid	Transverse wires			Longitudinal wires			Theoretical mass of sheet
		Quantity	Diameter	Distance	Quantity	Diameter	Distance	
(mm)			(mm)	(mm)		(mm)	(mm)	(Kg)
500 x 215	T131	33	5,0	150	15	5.0	150	21.5
500 x 215	T188	33	6,0	150	15	6.0	150	32.4
500 x 215	T196	50	5,0	100	22	5.0	100	33.5

Example: A T188 grid has square loops 150 mm x 150 mm with reinforcement Ø6/15 that corresponds to 188 mm²/m, similar in both directions. Its mass is 32.4 kg

TABLE 3

Distance s in mm	Area of reinforcement in mm ²							Pieces per m
	Ø 6mm	Ø 8mm	Ø 10mm	Ø 12mm	Ø 14mm	Ø 16mm	Ø 18mm	
60.0	471	838	1309	1885	2566	3352	4241	16.70
65.0	435	773	1208	1740	2368	3095	3915	15.40
70.0	404	718	1122	1616	2199	2873	3636	14.30
75.0	377	670	1047	1508	2052	2681	3393	13.40
80.0	353	628	982	1414	1924	2514	3181	12.50
85.0	333	591	924	1331	1811	2366	2994	11.80
90.0	314	559	873	1257	1710	2234	2828	11.10
95.0	298	529	827	1190	1620	2117	2679	10.50
100.0	283	500	785	1131	1539	2011	2545	10.00
105.0	269	479	748	1077	1466	1915	2424	9.50
110.0	257	457	714	1028	1399	1828	2314	9.10
115.0	246	437	683	984	1339	1749	2213	8.70
120.0	236	419	654	942	1283	1676	2121	8.30
125.0	226	402	628	905	1232	1609	2036	8.00
130.0	217	387	604	870	1184	1547	1958	7.70
135.0	209	372	582	838	1140	1490	1885	7.40
140.0	202	359	561	808	1100	1436	1818	7.10
145.0	195	347	542	780	1062	1387	1755	6.90
150.0	189	335	524	754	1026	1341	1697	6.70
155.0	182	324	507	730	993	1297	1642	6.50
160.0	177	314	491	707	962	1257	1590	6.30
165.0	171	305	476	685	933	1219	1542	6.10
170.0	166	296	462	665	905	1183	1497	5.90
175.0	162	287	449	646	879	1149	1454	5.70
180.0	157	279	436	628	855	1117	1414	5.60
185.0	153	272	425	611	832	1087	1376	5.40
190.0	149	265	413	595	810	1058	1339	5.30
195.0	145	258	403	580	789	1031	1305	5.10
200.0	141	251	393	565	769	1005	1272	5.00

Example: Ø12/15 corresponds to reinforcement area of 754 mm²/m. Equivalent reinforcement is Ø10/10 which corresponds to area of 785 mm²/m

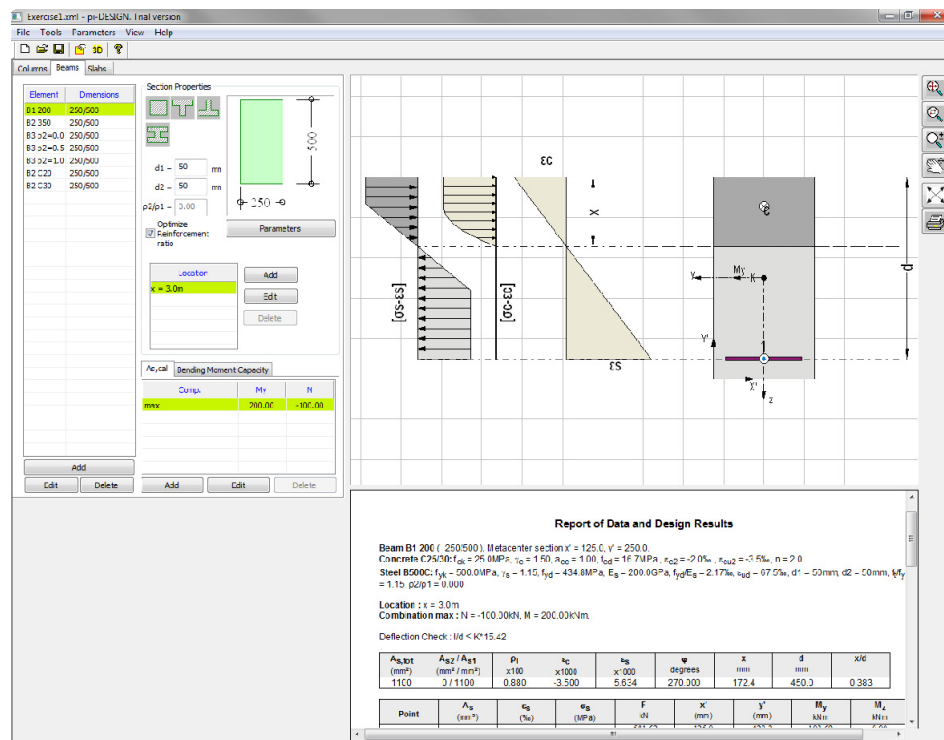
7. Applications

Exercise 1

Beam 'B1 200'

Calculate the required flexural reinforcement for the rectangular beam 250/500 under axial force load $N_d = -100$ kN and moment (a) $M_d = 200$ kNm and (b) $M_d = 350$ kNm. Material properties: C25, B500c, $d_1 = 50$ mm.

Using Pi-Design software package and file 'exercise1.xml', for beam 'B1 200' under moment load $M_d = 200$ kNm, we acquire the following solution:



The calculated required reinforcement is $A_{s,tot}=A_{s1}=1100 \text{ mm}^2$ [$\rho_1=0.880\%$, $\epsilon_c=-3.50\text{‰}$, $\epsilon_s=5.63\text{‰}$, $x/d=0.383$]. $A_{s2}=0$, means there is no need for compression reinforcement.

From table 1 we choose reinforcement 4Φ20 (1256 mm²).

In order to examine the inverse problem, we select the 'Calculate Bending Moment' tab. Setting the reinforcement we chose above

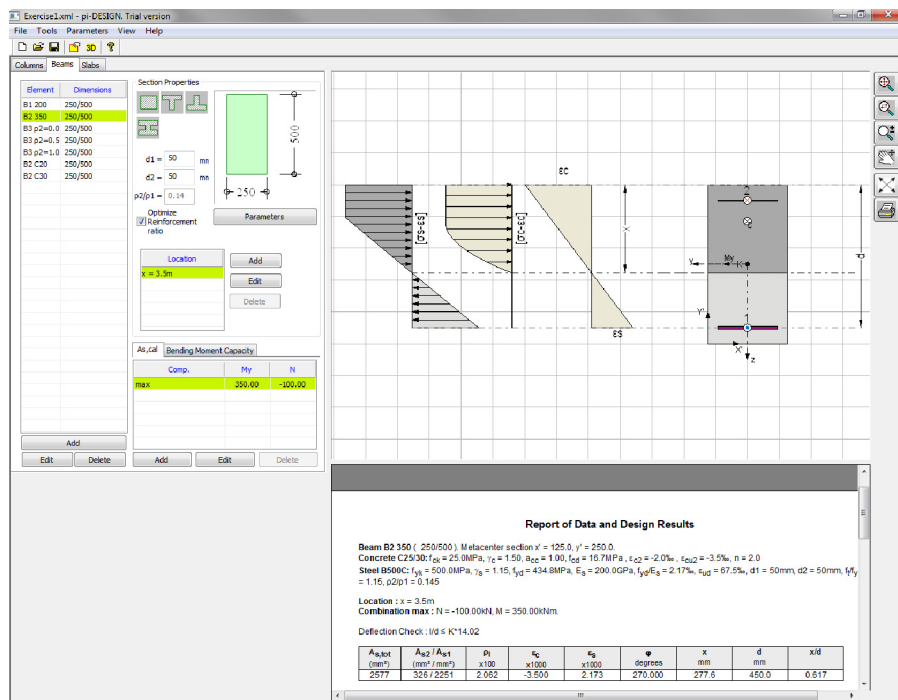
($A_{s1}=1256 \text{ mm}^2$), we get resisting moment $M_{Rd}=220.09 \text{ kNm}$ [$\epsilon_c=-3.50\text{‰}$, $\epsilon_s=4.69\text{‰}$, $x/d=0.428$]¹² as shown in the next figure.

Beam 'B2 350'

The same beam under moment load $M_d=350 \text{ kNm}$ and *with* the ρ_2/ρ_1 value defined automatically by checking the 'Optimize Reinforcement ratio' box, yields the following solution:

$A_{s,tot}=2577 \text{ mm}^2$ ($A_{s1}=2251 \text{ mm}^2$, $A_{s2}=326 \text{ mm}^2$) [$\rho_1=2.062\%$, $\epsilon_c=-3.50\text{‰}$, $\epsilon_s=2.17\text{‰}$, $x/d=0.617$]

¹² If we fill in the reinforcement field with $A_{s,eff}=1100 \text{ mm}^2$ that resulted from the design calculation, we would get $M_{Rd}=200.01 \text{ kNm}$ which is equal to M_d . The minimal variation from 200.00 kNm is due to the number of significant decimal digits used.



Notes:

Working on the same beam and altering some parameters we get the following results for comparison:

- In beam 'B3 $p_2/p_1=0.0$ ', a solution is requested for zero percentage of compression reinforcement. The result is: $A_{s,tot}=A_{s1}=7719$ mm² [$\rho_1=6.176\%$ (beyond the limits posed by all Codes), $\epsilon_c=-3.50\text{‰}$, $\epsilon_s=0.75\text{‰}$, $x/d=0.824$]
- In beam 'B3 $p_2/p_1=0.50$ ', a solution is requested for compression to tension percentage reinforcement ratio equal to 50.0% (as mandated by the Code at beam supports). The result is: $A_{s,tot}=2887$ mm² ($A_{s1}=1925$ mm², $A_{s2}=962$ mm²) [$\rho_1=2.310\%$, $\epsilon_c=-3.50\text{‰}$, $\epsilon_s=6.60\text{‰}$, $x/d=0.346$]
- In beam 'B3 $p_2/p_1=1.0$ ', a solution is requested for compression to tension percentage reinforcement ratio equal to 100%, i.e. symmetrical reinforcement (used in the case of seismic loading)

being the most unfavorable combination, e.g. in 'short' beams). The result is:

$A_{s,tot}=3671 \text{ mm}^2$ ($A_{s1}=1836 \text{ mm}^2$, $A_{s2}=1836 \text{ mm}^2$) [$\rho_l=2.937\%$, $\epsilon_c=-3.50\text{‰}$, $\epsilon_s=13.29\text{‰}$, $x/d=0.208$]

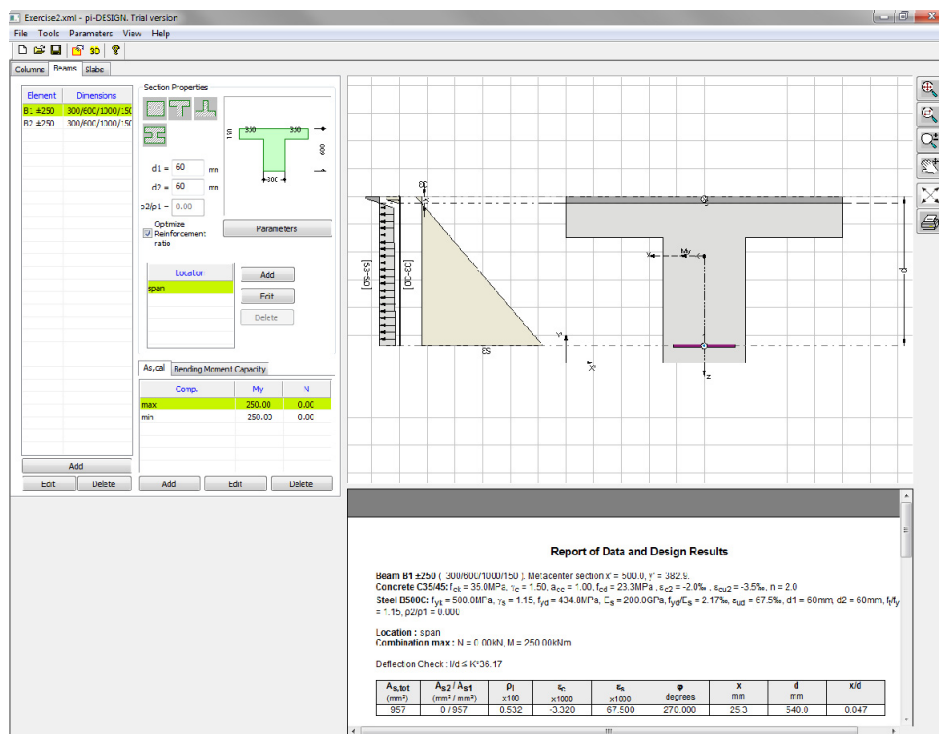
- In beam 'B3 C20' concrete C20 is selected in local parameters, with a ρ_2/ρ_1 value defined having the 'optimize reinforcement ratio' box checked. The result is: $A_{s,tot}=2866 \text{ mm}^2$ ($A_{s1}=2179 \text{ mm}^2$, $A_{s2}=686 \text{ mm}^2$) [$\rho_l=2.293\%$, $\epsilon_c=-3.50\text{‰}$, $\epsilon_s=2.17\text{‰}$, $x/d=0.617$]. $A_{s,tot}=2866 \text{ mm}^2$ for C20 concrete is greater than $A_{s,tot}=2577 \text{ mm}^2$ for C25 concrete.
- In beam 'B3 C30' concrete C30 is selected in local parameters, with a ρ_2/ρ_1 value defined by having the 'optimize reinforcement ratio' box checked. The result is: $A_{s,tot}=A_{s1}=2292 \text{ mm}^2$, $A_{s2}=0 \text{ mm}^2$ [$\rho_l=1.834\%$, $\epsilon_c=-3.50\text{‰}$, $\epsilon_s=2.31\text{‰}$, $x/d=0.602$]. $A_{s,tot}=2292 \text{ mm}^2$ for C30 concrete is significantly lower than $A_{s,tot}=2577 \text{ mm}^2$ for C25 concrete. Therefore the selection of C30 concrete is the most cost-effective of all.

Exercise 2

Beam 'B1 ±250'

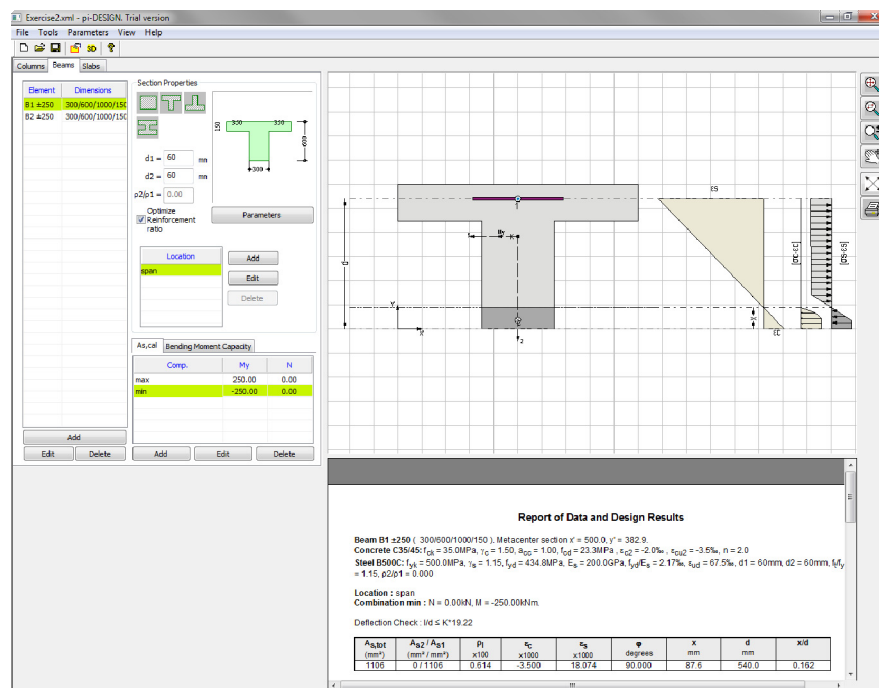
Calculate the required flexural reinforcement for T-beam with web 300/600 and flange 1000/150 under axial force load $N_d=0$ and moment (a) $M_d=+250$ kNm and (b) $M_d=-250$ kNm. Material properties: C35, B500c, $d_1=d_2=60$ mm.

Utilizing Pi-Design software package and file 'exercise2.xml', for beam 'B1 ±250' at 'max' combination with moment load $M_d=250$ kNm, we get the following solution:



The required reinforcement is $A_{s1}=957 \text{ mm}^2$ [$\rho_l=0.532\%$, $\epsilon_c=-3.32\text{‰}$, $\epsilon_s=67.50\text{‰}$, $x/d=0.047$].

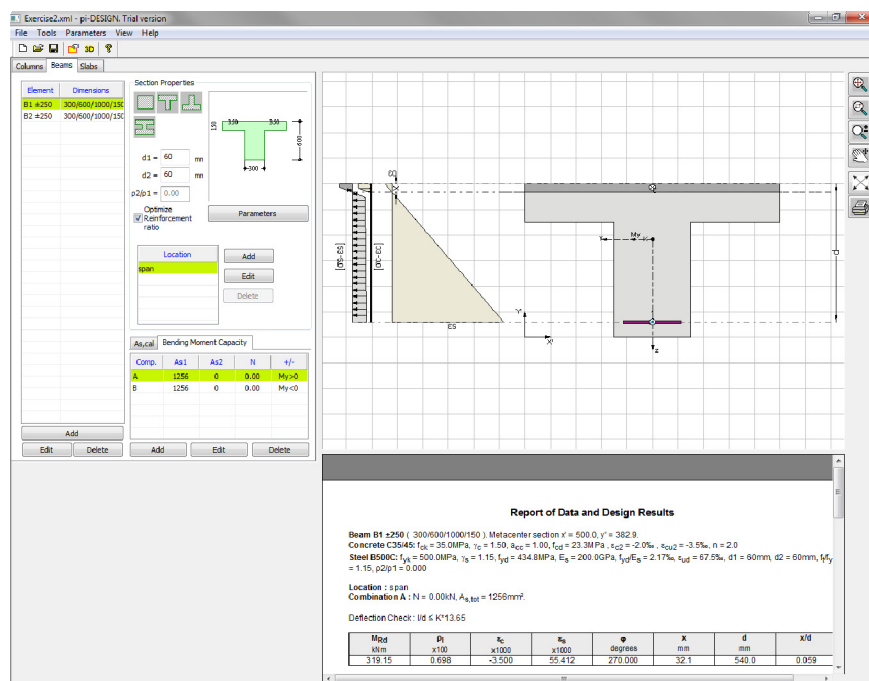
Examining combination 'min' with a moment load $M_d=-250 \text{ kNm}$, we get the following solution:



The required reinforcement is $A_{s1}=1106 \text{ mm}^2$ [$\rho_l=0.614\%$, $\epsilon_c=-3.50\text{‰}$, $\epsilon_s=18.07\text{‰}$, $x/d=0.162$].

Now let's assume we choose to place the same amount of reinforcement $4\Phi 20$ (1256 mm^2) in the upper and lower fibers of the beam.

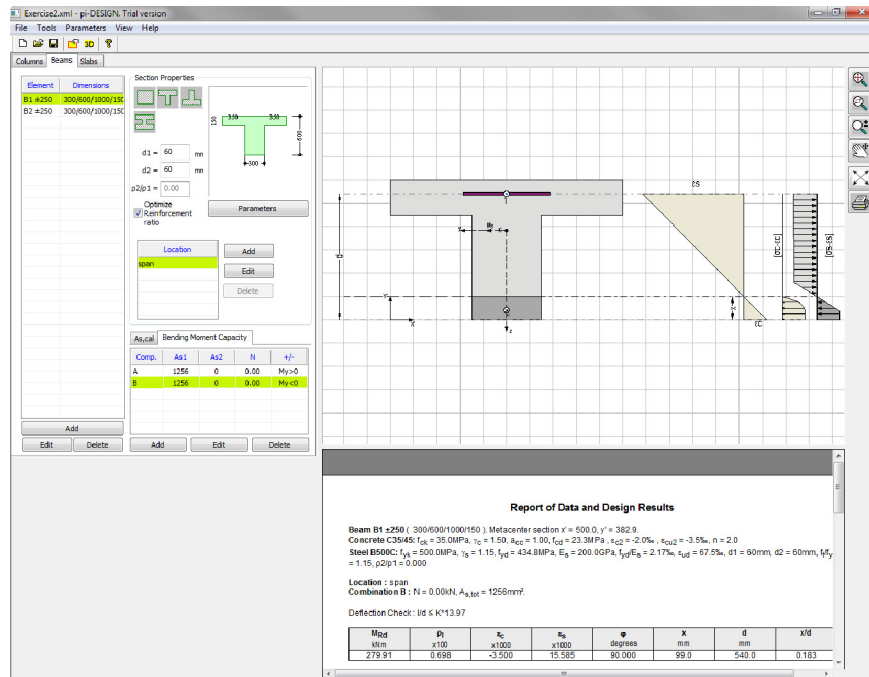
In the 'Calculate Bending Moment' tab we set $A_{s1}=1256 \text{ mm}^2$ in the first combination and we request the positive resisting moment. The solution is presented below:



The positive resisting moment is $M_{Rd}=319.15 \text{ kNm}$ [$\rho_l=0.698\%$, $\epsilon_c=-3.50\text{‰}$,

$\epsilon_s=55.41\text{‰}$, $x/d=0.059$].

In the second combination we set $A_{s1}=1256 \text{ mm}^2$ again, but now we request the negative resisting moment. The solution is as follows:



The negative resisting moment is $M_{Rd}=279.91 \text{ kNm}$ [$\rho_l=0.698\%$, $\epsilon_c=-3.50\%$, $\epsilon_s=15.59\%$, $x/d=0.183$].

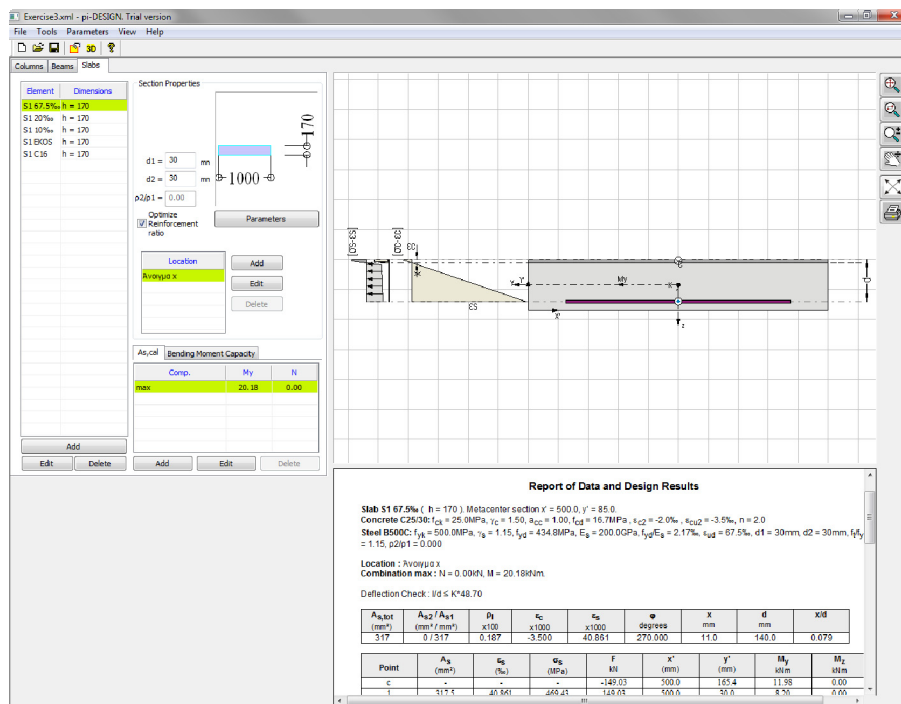
Beam 'B2 ±250' has the same dimensions with beam 'B1 ±250' but is inverted. For the same loads and reinforcement the results are relative.

Exercise 3

Slab 'S1 67.5‰'

Calculate the required flexural reinforcement for the given slab. Material properties: C25, B500c, $d_1=30$ mm. The design moment is $M_d=20.18$ kNm.

Using Pi-Design software package and file exercise3.xml, we get the following solution:

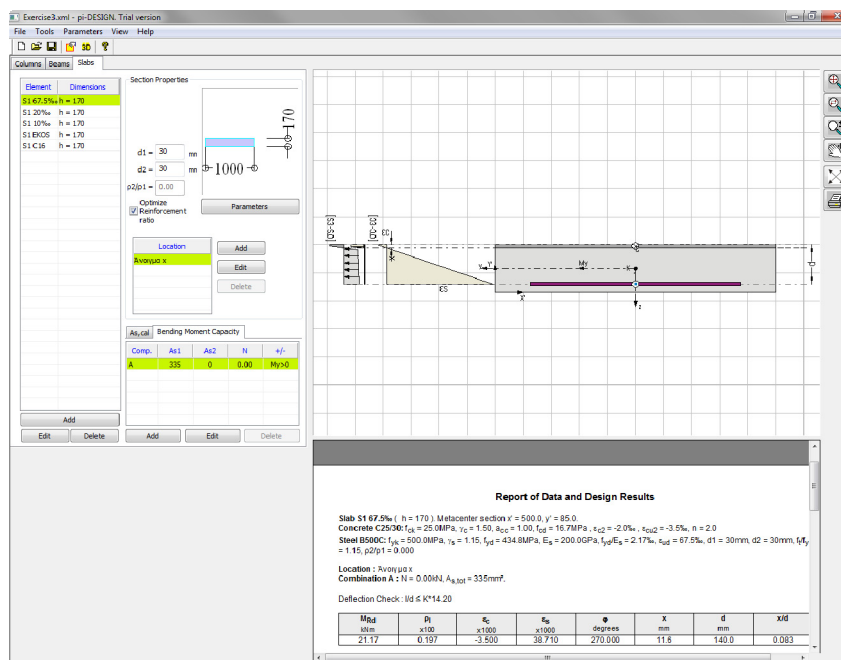


The required reinforcement is $A_{s,tot}=A_{s1}=3175 \text{ mm}^2$ [$\rho_l=0.187\%$, $\epsilon_c=-3.50\%$,

$\epsilon_s=40.86\%$, $x/d=0.079$]

From table 3 we choose reinforcement $\Phi 8/15$ (335 mm^2)

Examining the inverse problem 'B' we select 'Calculate Bending Moment' and give $A_{s1}=335 \text{ mm}^2$. The calculated resisting moment is $M_{Rd}=21.17 \text{ kNm}$ [$\rho_l=0.197\%$, $\epsilon_c=-3.50\%$, $\epsilon_s=38.71\%$, $x/d=0.083$] as presented on the following figure.



If we set the value of the reinforcement field equal to 3175 mm^2 , which was the design calculation result, Pi-Design would yield $M_d=20.18 \text{ kNm}$.

Notes:

Working with the same slab and altering some parameters we receive the following results for comparison:

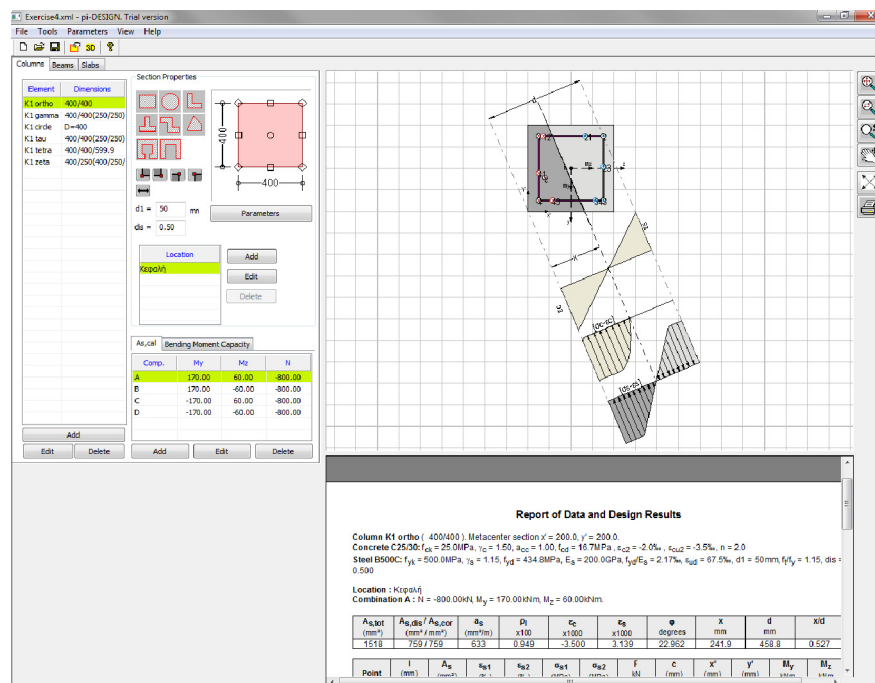
- For slab 'S1 20%', altering local parameters: $\varepsilon_{ud}=20\text{‰}$, the respective design results are: $A_{s1}=332 \text{ mm}^2$ [$\rho_l=0.195\%$, $\varepsilon_c=-2.08\text{‰}$, $\varepsilon_s=20.00\text{‰}$, $x/d=0.094$]
- For slab 'S1 10%', altering local parameters: $\varepsilon_{ud}=10\text{‰}$, the respective design results are: $A_{s1}=341 \text{ mm}^2$ [$\rho_l=0.201\%$, $\varepsilon_c=-1.38\text{‰}$, $\varepsilon_s=10.00\text{‰}$, $x/d=0.121$]
- For slab 'S1 EKOS', altering local parameters: EKOS2000 (with the main differences being $a_{cc}=0.85$ and $\varepsilon_{ud}=10\text{‰}$), the respective results are: $A_{s1}=348 \text{ mm}^2$ [$\rho_l=0.205\%$, $\varepsilon_c=-1.54\text{‰}$, $\varepsilon_s=10.00\text{‰}$, $x/d=0.133$]
- For slab 'S1 C16', altering local parameters: EKOS2000, C16 and S400 (in accordance with exercise 6 of the old book), the respective results are: $A_{s1}=444 \text{ mm}^2$ [$\rho_l=0.261\%$, $\varepsilon_c=-2.15\text{‰}$, $\varepsilon_s=10.00\text{‰}$, $x/d=0.177$]

Exercise 4

Column 'C1 ortho'

Calculate the required reinforcement for column with square cross-section 400/400 under biaxial bending for the 4 loading combinations $N_d = -800$ kN, $M_{yd} = \pm 170$ kNm and $M_{zd} = \pm 60$ kNm. Material properties: C25, B500c, $d_1 = 50$ mm. Then, calculate the required reinforcement for respective Gamma-shaped, Circular, T-shaped, Quadrangular and Z-shaped cross-sections for the same loading.

Utilizing Pi-Design software package and file 'exercise4.xml', the given column is designed for the 4 load combinations.



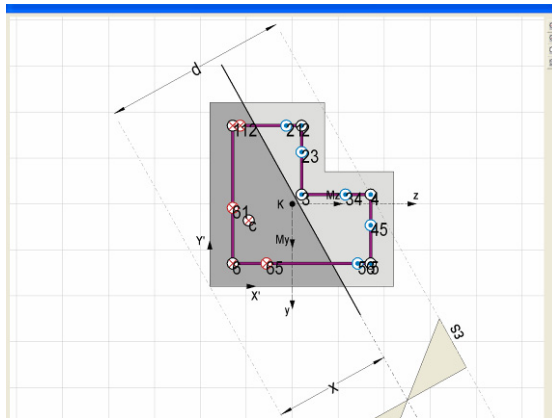
In the first combination (both moments are positive) biaxial flexure yields a maximum compressive stress at the lower left corner of the column while the neutral axis has an angle $\varphi=22.96^\circ$ towards y axis and its distance from the upper fiber of the cross-section is $x=241.9$ mm.

The total required reinforcement is $A_{s,tot}=1518$ mm² ($A_{s,cor}=759$ mm² and $A_{s,dis}=760$ mm²) [$\rho_l=0.949\%$, $\varepsilon_c=-3.50\text{‰}$, $\varepsilon_s=3.14\text{‰}$, $\varphi=22.96^\circ$, $x=242$ mm]

Due to the double symmetry of the cross-section and the symmetry of the loads, all results (except for angle φ) on the other 3 combinations are the same.

Next, the other 5 column types are examined:

Column 'C1 gamma' 400/400(250/250)



Combination A

$$N_d = -800 \text{ kN},$$

$$M_{yd} = 170 \text{ kNm}$$

$$M_{zd} = 60 \text{ kNm}$$

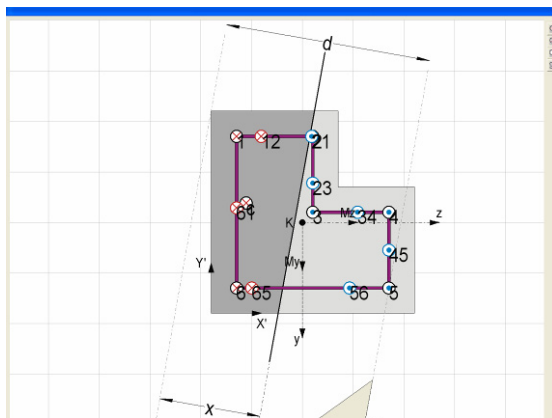
$$A_{s,tot} = 3325 \text{ mm}^2$$

$$A_{s,cor}/A_{s,dis} = 1663/1663$$

$$[\rho_f = 2.418\%, \epsilon_c = -3.50\text{‰},$$

$$\epsilon_s = 2.00\text{‰},$$

$$\varphi = 28.98^\circ, x = 256 \text{ mm}]$$



Combination B

$$N_d = -800 \text{ kN},$$

$$M_{yd} = 170 \text{ kNm}$$

$$M_{zd} = -60 \text{ kNm}$$

$$A_{s,tot} = 1957 \text{ mm}^2$$

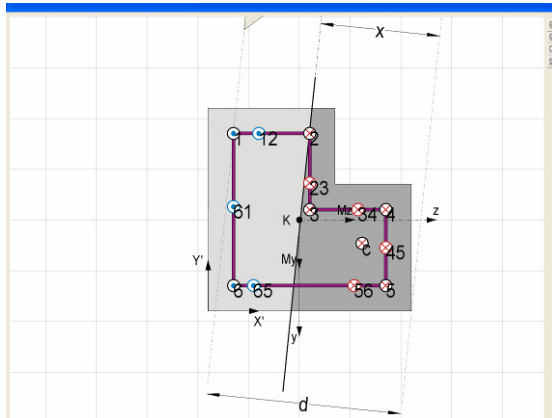
$$A_{s,cor}/A_{s,dis} = 978/978$$

$$[\rho_f = 1.423\%, \epsilon_c = -3.50\text{‰},$$

$$\epsilon_s = 3.61\text{‰},$$

$$\varphi = 349.94^\circ \text{ (or } -10.06^\circ),$$

$$x = 200 \text{ mm}]$$



Combination C

$$N_d = -800 \text{ kN},$$

$$M_{yd} = -170 \text{ kNm} \quad M_{zd} = 60 \text{ kNm}$$

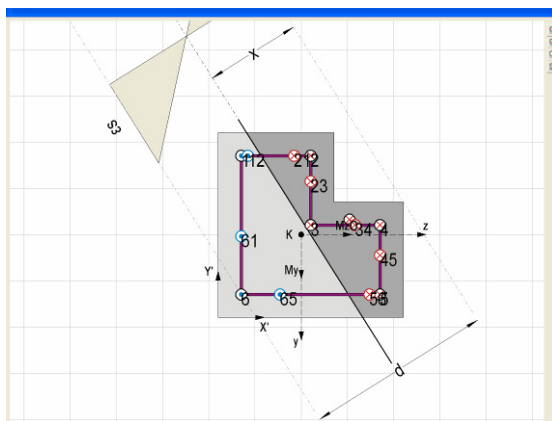
$$A_{s,tot} = 1905 \text{ mm}^2$$

$$A_{s,cor}/A_{s,dis} = 953/953$$

$$[\rho_l = 1.385\%, \quad \varepsilon_c = -3.50\text{‰},$$

$$\varepsilon_s = 2.21\text{‰},$$

$$\varphi = 174.13^\circ, \quad x = 236 \text{ mm}]$$



Combination D

$$N_d = -800 \text{ kN},$$

$$M_{yd} = -170 \text{ kNm}$$

$$M_{zd} = -60 \text{ kNm}$$

$$A_{s,tot} = 3271 \text{ mm}^2$$

$$A_{s,cor}/A_{s,dis} = 1635/1635$$

$$[\rho_l = 2.378\%, \quad \varepsilon_c = -3.50\text{‰},$$

$$\varepsilon_s = 3.31\text{‰},$$

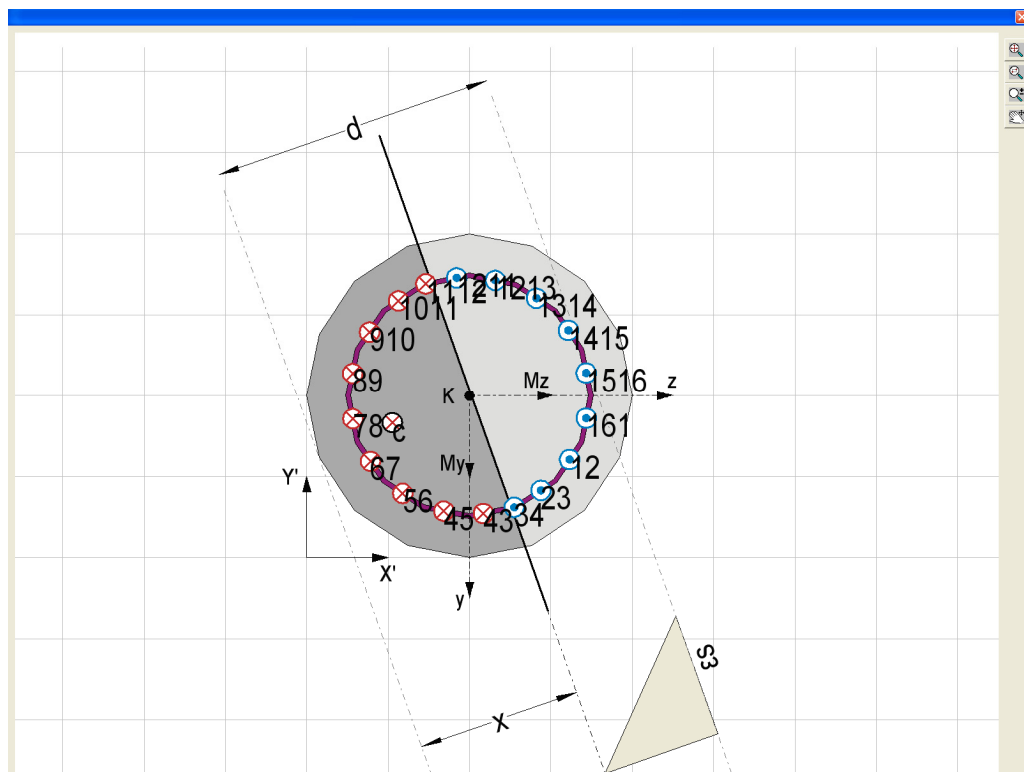
$$\varphi = 212.11^\circ, \quad x = 207 \text{ mm}]$$

Notes:

- We observe that the most unfavorable combinations are A and D (because of the smaller lever arm between reinforcing bars placed at the corners of the column).
- For concrete strength class C30 the highest reinforcement demand would be $A_{s,tot} = 2938 \text{ mm}^2$ and for class C35 concrete would be $A_{s,tot} = 2656 \text{ mm}^2$ (against 3271 mm^2 for C25).

Column 'C1 circle' $D=400$

Because of the absolute symmetry of the cross-section all combinations yield the same solution, so only combination A is examined.



$$N_d = -800 \text{ kN}, M_{yd} = 270 \text{ kNm}, M_{zd} = 60 \text{ kNm}$$

$$A_{s,tot} = 2893 \text{ mm}^2, A_{s,dis}/A_{s,cor} = 2893/0.0$$

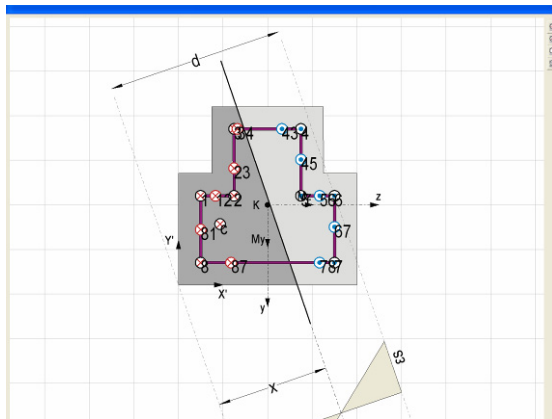
$[\rho_l=2.362\%, \varepsilon_c=-3.50\text{‰}, \varepsilon_s=2.54\text{‰},$

$\varphi=19.44^\circ, x=202 \text{ mm}]$

Notes:

- Reinforcement of the circular cross-section is assumed evenly distributed (dis=1.0). Circular cross-section is simulated by the program as a normal polygon (a 16-sided one for the specific case). If we alter the number of sides of the element to 32 then the result becomes $A_{s,tot}=2764 \text{ mm}^2$
- For concrete strength class C30 the required reinforcement would be $A_{s,tot}=2480 \text{ mm}^2$ and for strength class C35 would be $A_{s,tot}=2168 \text{ mm}^2$ (against 2893 mm^2 for C25).

Column 'C1 tau' 400/400(250/250)



Combination A

$N_d = -800 \text{ kN}$,

$M_{yd} = 170 \text{ kNm}$

$M_{zd} = 60 \text{ kNm}$

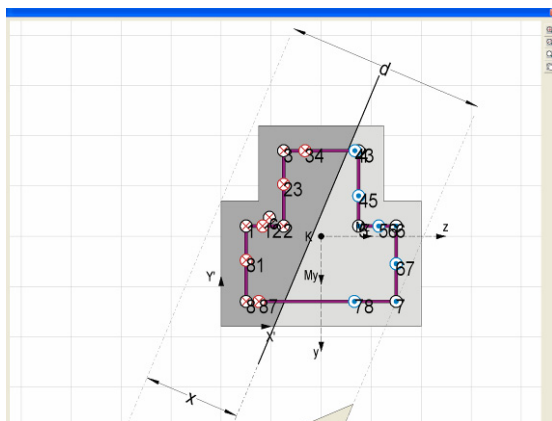
$A_{s,tot} = 2661 \text{ mm}^2$

$A_{s,cor}/A_{s,dis} = 1330/1330$

$[\rho] = 1.935\%$, $\epsilon_c = -3.50\text{‰}$,

$\epsilon_s = 1.99\text{‰}$,

$\varphi = 18.84^\circ$, $x = 252 \text{ mm}$



Combination B

$N_d = -800 \text{ kN}$,

$M_{yd} = 170 \text{ kNm}$

$M_{zd} = -60 \text{ kNm}$

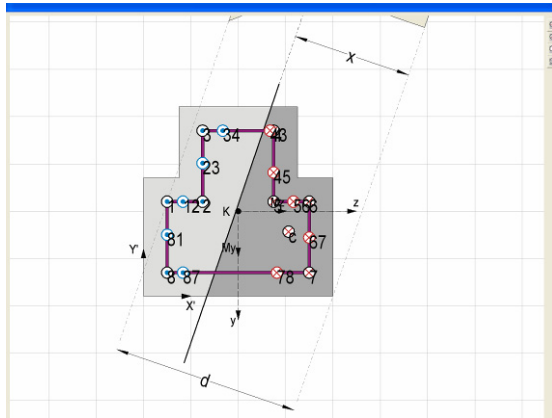
$A_{s,tot} = 2496 \text{ mm}^2$

$A_{s,cor}/A_{s,dis} = 1248/1248$

$[\rho] = 1.815\%$, $\epsilon_c = -3.50\text{‰}$,

$\epsilon_s = 3.70\text{‰}$,

$\varphi = 337.16^\circ$, $x = 195 \text{ mm}$



Combination C

$N_d = -800 \text{ kN}$,

$M_{yd} = -170 \text{ kNm}$

$M_{zd} = 60 \text{ kNm}$

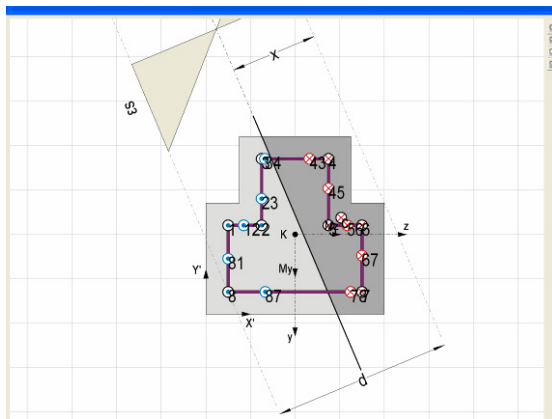
$A_{s,tot} = 2660 \text{ mm}^2$

$A_{s,cor}/A_{s,dis} = 1330/1330$

$[\rho_f = 1.935\%, \epsilon_c = -3.50\text{‰},$

$\epsilon_s = 1.99\text{‰},$

$\varphi = 161.18^\circ, x = 252 \text{ mm}]$



Combination D

$N_d = -800 \text{ kN}$,

$M_{yd} = -170 \text{ kNm}$

$M_{zd} = -60 \text{ kNm}$

$A_{s,tot} = 2497 \text{ mm}^2$

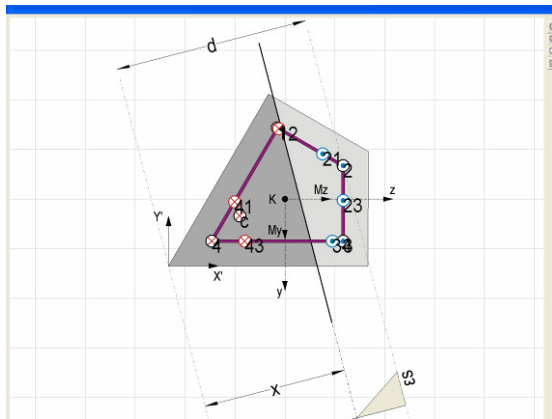
$A_{s,cor}/A_{s,dis} = 1248/1248$

$[\rho_f = 1.816\%, \epsilon_c = -3.50\text{‰},$

$\epsilon_s = 3.70\text{‰},$

$\varphi = 202.86^\circ, x = 195 \text{ mm}]$

Column 'C1 tetra' 400/400/60°



Combination A

$$N_d = -800 \text{ kN},$$

$$M_{yd} = 170 \text{ kNm}$$

$$M_{zd} = 60 \text{ kNm}$$

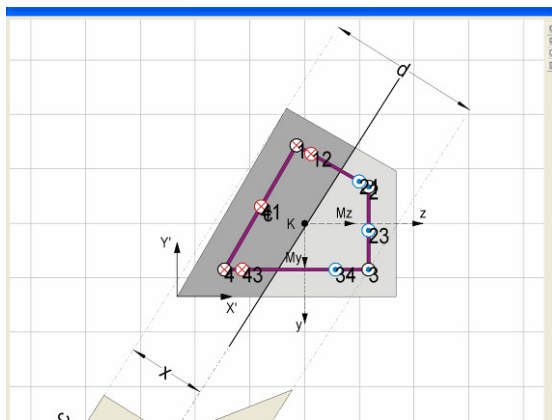
$$A_{s,tot} = 5025 \text{ mm}^2$$

$$A_{s,cor}/A_{s,dis} = 2513/2513$$

$$[\rho] = 5.440\%, \epsilon_c = -3.50\text{‰},$$

$$\epsilon_s = 1.23\text{‰},$$

$$\varphi = 14.54^\circ, x = 288 \text{ mm}]$$



Combination B

$$N_d = -800 \text{ kN},$$

$$M_{yd} = 170 \text{ kNm}$$

$$M_{zd} = -60 \text{ kNm}$$

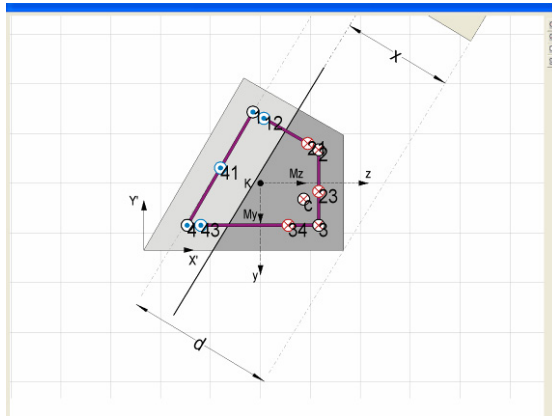
$$A_{s,tot} = 3746 \text{ mm}^2$$

$$A_{s,cor}/A_{s,dis} = 1873/1873$$

$$[\rho] = 4.056\%, \epsilon_c = -3.50\text{‰},$$

$$\epsilon_s = 3.39\text{‰},$$

$$\varphi = 327.71^\circ, x = 145 \text{ mm}]$$



Combination C

$$N_d = -800 \text{ kN},$$

$$M_{yd} = -170 \text{ kNm}$$

$$M_{zd} = 60 \text{ kNm}$$

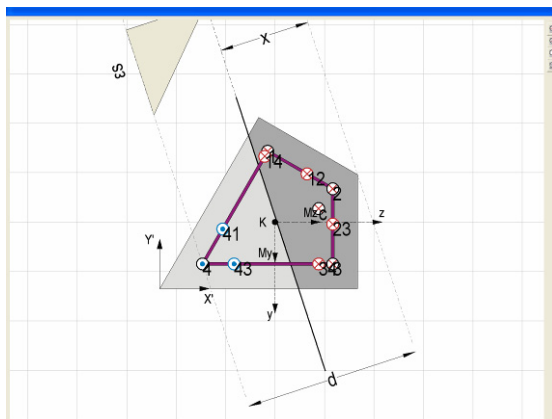
$$A_{s,tot} = 5114 \text{ mm}^2$$

$$A_{s,cor}/A_{s,dis} = 2557/2557$$

$$[\rho_l = 5.536\%, \epsilon_c = -3.50\text{‰},$$

$$\epsilon_s = 1.18\text{‰},$$

$$\varphi = 148.86^\circ, x = 224 \text{ mm}]$$



Combination D

$$N_d = -800 \text{ kN},$$

$$M_{yd} = 170 \text{ kNm}$$

$$M_{zd} = 60 \text{ kNm}$$

$$A_{s,tot} = 3646 \text{ mm}^2$$

$$A_{s,cor}/A_{s,dis} = 1823/1823$$

$$[\rho_l = 3.948\%, \epsilon_c = -3.50\text{‰},$$

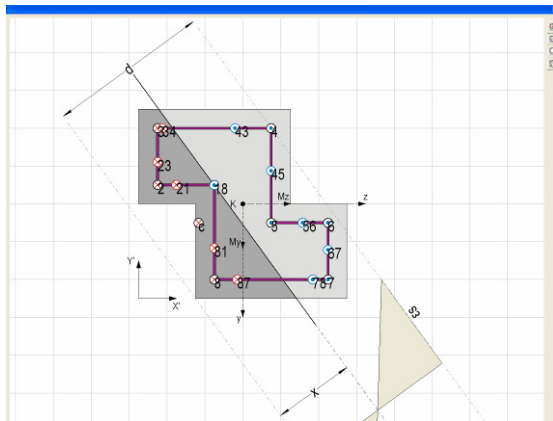
$$\epsilon_s = 3.20\text{‰},$$

$$\varphi = 198.05^\circ, x = 1850 \text{ mm}]$$

Notes:

- The most unfavorable combination is C. We could see this combination directly, if we select 'Tools' \ 'Calc Max As'.
- The calculated reinforcement percentage demand is very large and therefore a change in member dimensions is required, e.g. for a cross-section with dimensions 450/450/60° the required reinforcement drops to $A_{s,tot} = 3366 \text{ mm}^2$ ($\rho_l = 2.88\%$) from $A_{s,tot} = 5114 \text{ mm}^2$ ($\rho_l = 5.54\%$).

Column 'C1 zeta' 400/250/(400/250/250/250)



Combination A

$N_d = -800 \text{ kN}$,

$M_{yd} = 170 \text{ kNm}$

$M_{zd} = 60 \text{ kNm}$

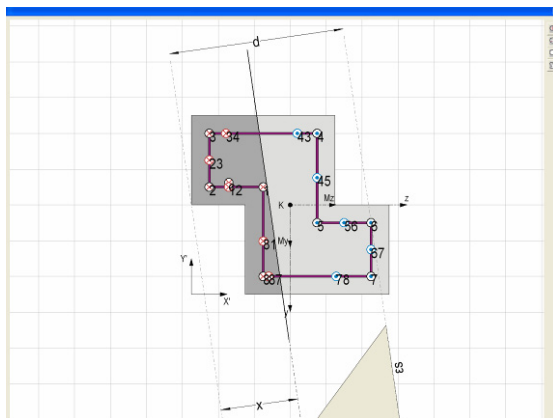
$A_{s,tot} = 1687 \text{ mm}^2$

$A_{s,cor}/A_{s,dis} = 843/843$

$[\rho_l = 0.843\%, \epsilon_c = -3.50\text{‰}]$,

$\epsilon_s = 3.42\text{‰}$,

$\varphi = 36.04^\circ, x = 216 \text{ mm}]$



Combination B

$N_d = -800 \text{ kN}$,

$M_{yd} = 170 \text{ kNm}$

$M_{zd} = -60 \text{ kNm}$

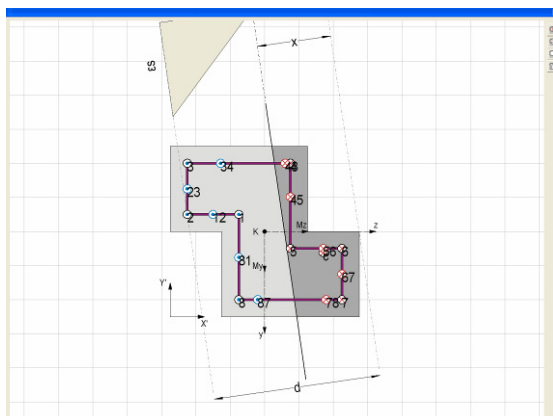
$A_{s,tot} = 495 \text{ mm}^2$

$A_{s,cor}/A_{s,dis} = 247/247$

$[\rho_l = 0.247\%, \epsilon_c = -3.50\text{‰}]$,

$\epsilon_s = 4.43\text{‰}$,

$\varphi = 8.20^\circ, x = 215 \text{ mm}]$



Combination C

$N_d = -800 \text{ kN}$,

$M_{yd} = -170 \text{ kNm}$

$M_{zd} = 60 \text{ kNm}$

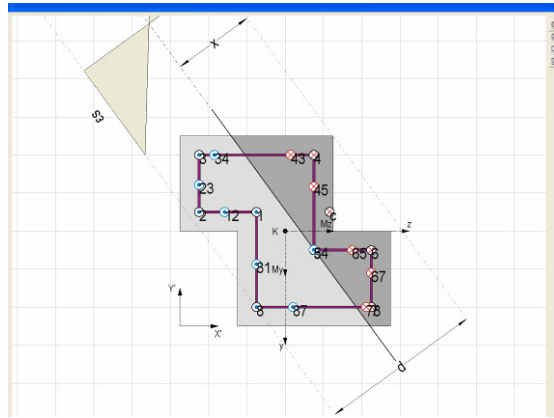
$A_{s,tot} = 495 \text{ mm}^2$

$A_{s,cor}/A_{s,dis} = 247/247$

$[\rho_l = 0.247\%, \epsilon_c = -3.50\text{‰}]$,

$\epsilon_s = 4.43\text{‰}$,

$\varphi = 188.20^\circ, x = 215 \text{ mm}]$



Combination D

$$N_d = -800 \text{ kN},$$

$$M_{yd} = 170 \text{ kNm}$$

$$M_{zd} = 60 \text{ kNm}$$

$$A_{s,tot} = 1687 \text{ mm}^2$$

$$A_{s,cor}/A_{s,dis} = 843/843$$

$$[\rho_f = 0.843\%, \epsilon_c = -3.50\%,$$

$$\epsilon_s = 3.42\%,$$

$$\varphi = 216.04^\circ, x = 216 \text{ mm}]$$

Note:

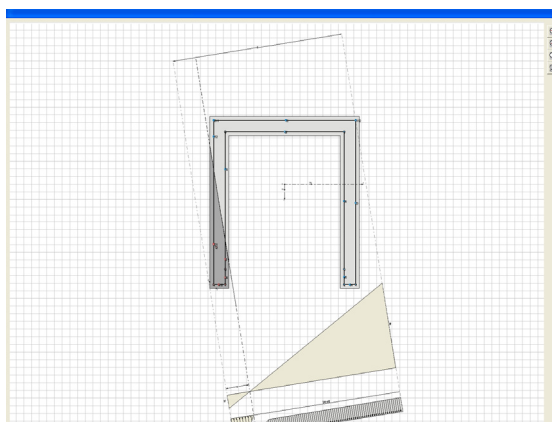
- The above Z-shaped cross-section is not inscribed in the 400x400 square like all the previous sections, therefore its calculated reinforcement is not comparable to the calculated reinforcement of the other sections.

Exercise 5

Elevator core 'L1 pi'

Calculate the required reinforcement of the elevator core (see figure below) under biaxial bending for the 4 load combinations $N_d=-3000$ kN, $M_{yd}=-170$ kNm $M_{yd}=\pm 5000$ kNm and $M_{zd}=\pm 4000$ kNm. Material properties C35, B500c, $d_1=50$ mm.

Using Pi-Design software package and file 'exercise5.xml', the elevator core is solved for the 4 combinations.



Combination A

$N_d=-3000$ kN,

$M_{yd}=5000$ kNm

$M_{zd}=4000$ kNm

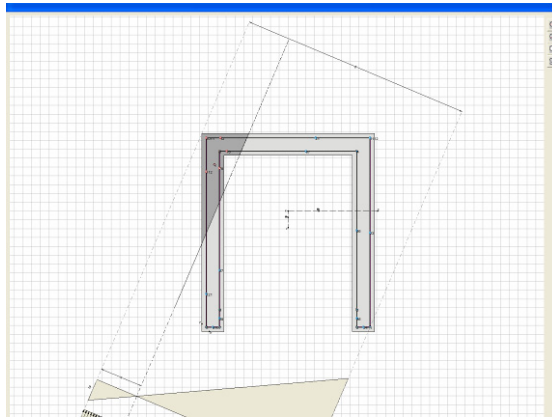
$A_{s,tot}=5939$ mm²

$A_{s,cor}/A_{s,dis}=2970/2970$

$[\rho_f=0.399\%, \epsilon_c=-3.500\%,$

$\epsilon_s=22.600\%, \varphi=9.129^\circ,$

$x=398$ mm]



Combination B

$N_d = -3000 \text{ kN}$,

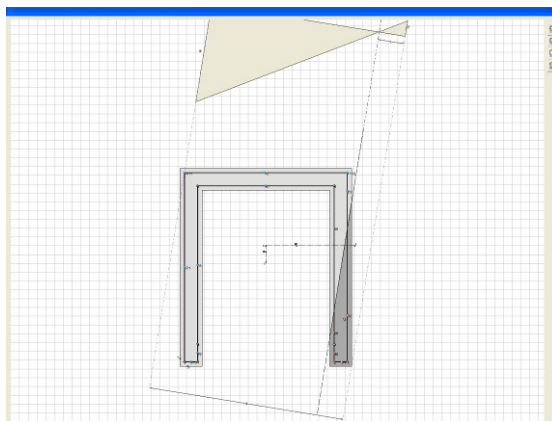
$M_{yd} = 5000 \text{ kNm}$

$M_{zd} = -4000 \text{ kNm}$

$A_{s,tot} = 6778 \text{ mm}^2$

$A_{s,cor}/A_{s,dis} = 3389/3389$

$[\rho_l = 0.456\%, \epsilon_c = -3.500\text{‰},$
 $\epsilon_s = 15.324\text{‰}, \varphi = 337.615^\circ,$
 $x = 482 \text{ mm}]$



Combination C

$N_d = -3000 \text{ kN}$,

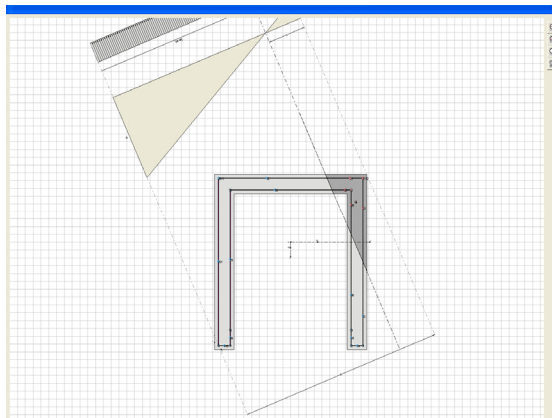
$M_{yd} = -5000 \text{ kNm}$

$M_{zd} = 4000 \text{ kNm}$

$A_{s,tot} = 5939 \text{ mm}^2$

$A_{s,cor}/A_{s,dis} = 2970/2970$

$[\rho_l = 0.399\%, \epsilon_c = -3.500\text{‰},$
 $\epsilon_s = 22.600\text{‰},$
 $\varphi = 170.871^\circ, x = 298 \text{ mm}]$



Combination D

$N_d = -3000 \text{ kN}$,

$M_{yd} = -5000 \text{ kNm}$

$M_{zd} = -4000 \text{ kNm}$

$A_{s,tot} = 6778 \text{ mm}^2$

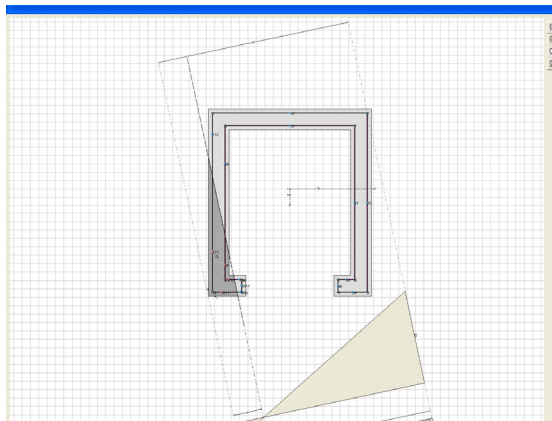
$A_{s,cor}/A_{s,dis} = 3389/3389$

$[\rho_l = 0.456\%, \epsilon_c = -3.500\text{‰},$
 $\epsilon_s = 15.324\text{‰},$
 $\varphi = 202.385^\circ, x = 482 \text{ mm}]$

Notes:

- If we assume only corner reinforcements ($\text{dis}=0$), the 4 combinations yield reinforcement demands $A_{s,\text{tot}}=6157 \text{ mm}^2$, 6138 mm^2 , 6157 mm^2 , 6138 mm^2 respectively.

In the following, the same elevator core with small protrusions added at its corners is designed for the same loads.

Elevator core 'L1 lift'

Combination A

$$N_d = -3000 \text{ kN},$$

$$M_{yd} = 5000 \text{ kNm}$$

$$M_{zd} = 4000 \text{ kNm}$$

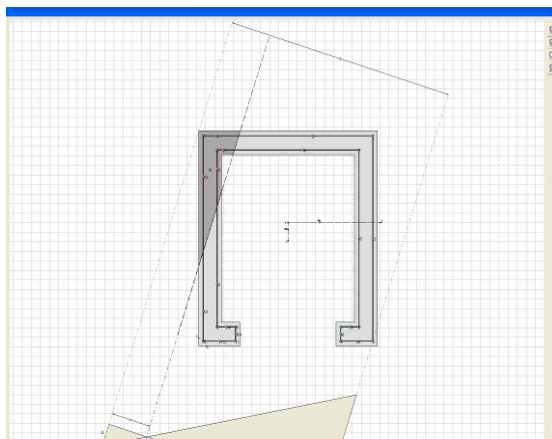
$$A_{s,\text{tot}} = 6227 \text{ mm}^2$$

$$A_{s,\text{cor}}/A_{s,\text{dis}} = 3114/3114$$

$$[\rho] = 0.392\%, \quad \varepsilon_c = -3.500\text{‰},$$

$$\varepsilon_s = 19.898\text{‰},$$

$$\varphi = 11.806^\circ, \quad x = 346 \text{ mm}]$$



Combination B

$$N_d = -3000 \text{ kN},$$

$$M_{yd} = 5000 \text{ kNm}$$

$$M_{zd} = -4000 \text{ kNm}$$

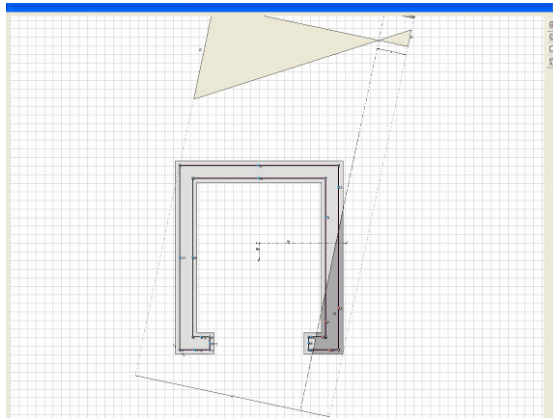
$$A_{s,\text{tot}} = 6492 \text{ mm}^2$$

$$A_{s,\text{cor}}/A_{s,\text{dis}} = 3246/3246$$

$$[\rho] = 0.409\%, \quad \varepsilon_c = -3.500\text{‰},$$

$$\varepsilon_s = 16.679\text{‰},$$

$$\varphi = 342.490^\circ, \quad x = 429 \text{ mm}]$$



Combination C

$$N_d = -3000 \text{ kN},$$

$$M_{yd} = -5000 \text{ kNm}$$

$$M_{zd} = 4000 \text{ kNm}$$

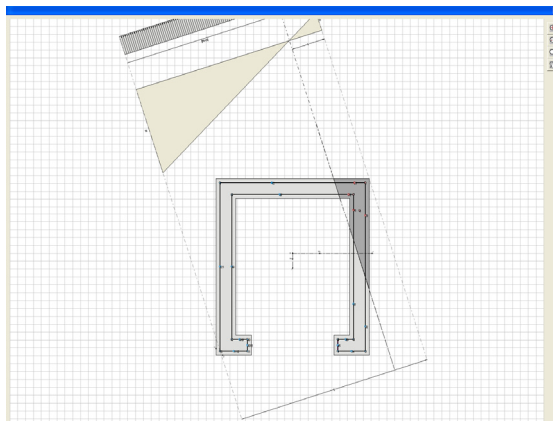
$$A_{s,tot} = 6227 \text{ mm}^2$$

$$A_{s,cor}/A_{s,dis} = 3114/3114$$

$$[\rho_l = 0.392\%, \epsilon_c = -3.500\%,$$

$$\epsilon_s = 19.898\%,$$

$$\varphi = 11.806^\circ, x = 346 \text{ mm}]$$



Combination D

$$N_d = -3000 \text{ kN},$$

$$M_{yd} = -5000 \text{ kNm}$$

$$M_{zd} = -4000 \text{ kNm}$$

$$A_{s,tot} = 6492 \text{ mm}^2$$

$$A_{s,cor}/A_{s,dis} = 3246/3246$$

$$[\rho_l = 0.409\%, \epsilon_c = -3.500\%,$$

$$\epsilon_s = 16.679\%,$$

$$\varphi = 342.490^\circ, x = 429 \text{ mm}]$$

Notes:

- If we assume only corner reinforcement (dis=0), the 4 combinations yield reinforcement demands $A_{s,tot} = 6560 \text{ mm}^2$, 6025 mm^2 , 6560 mm^2 , 6025 mm^2 respectively.
- If we assume only distributed reinforcement (dis=1), the 4 combinations yield reinforcement demands $A_{s,tot} = 5949 \text{ mm}^2$, 7446 mm^2 , 5949 mm^2 , 7446 mm^2 respectively.

